基础光学

手写笔记

参宿四星云

Betelgeuse Nebula

2022年12月 完稿
2023年10月 整理

~~参考~~ 抄的文献

[1] 陈敏,赵福利,董建文.光学
[2] 赵凯华.光学
[3] Eugene Hecht. OPTICS

~~参考~~ 抄的课件/课程

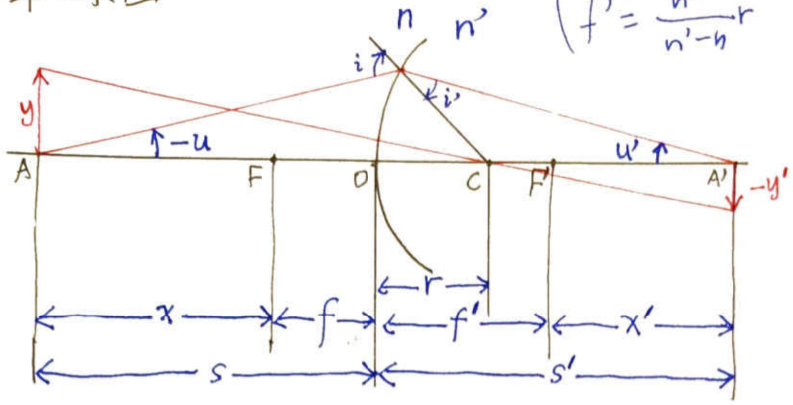
[1] 中大物院 陈瑞《光学》
[2] 中大物天 叶贤基《波动光学》

Part 1

同步课程

陈瑞《光学》

1° 单球面



$$\begin{cases} f=\dfrac{n}{n'-n}r \\ f'=\dfrac{n'}{n'-n}r \end{cases}$$

← 正弦定理 × 近轴近似:

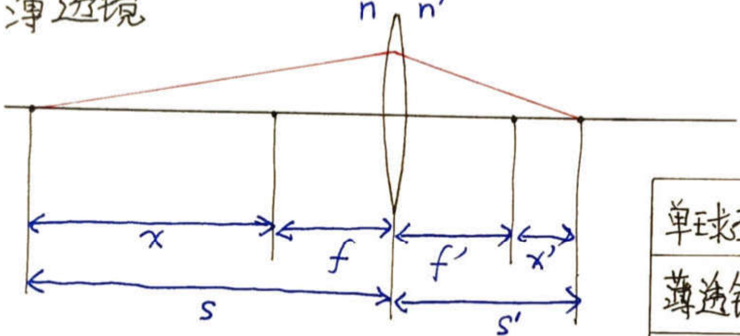
1° 物像位置关系公式 $\dfrac{n'}{s'}+\dfrac{n}{s}=\dfrac{n'-n}{r}$

1°2°3°
$$\begin{cases} \text{高斯公式} & \dfrac{f}{s}+\dfrac{f'}{s'}=1 \\ \text{牛顿公式} & xx'=ff' \\ \text{垂轴放大率} & \beta\equiv\dfrac{y'}{y}=-\dfrac{x'}{f'}=-\dfrac{f}{x} \\ \text{轴向放大率} & \alpha\equiv\dfrac{ds'}{ds}=-\dfrac{n'}{n}\beta^2 \\ \text{角放大率} & \gamma\equiv\dfrac{u'}{u}=-\dfrac{s}{s'} \end{cases}$$

$$\Longrightarrow \begin{cases} \alpha\gamma=\beta \\ \beta\gamma=\dfrac{f}{f'}=\dfrac{n}{n'} \end{cases}$$

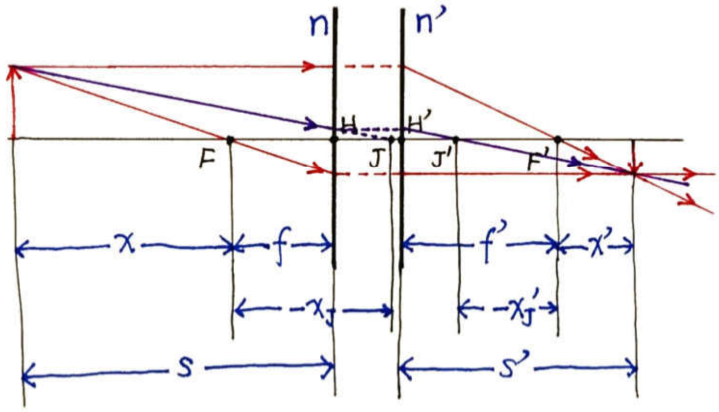
1° 拉赫不变量 $J\equiv nyu=n'y'u'$

2° 薄透镜



	光学系统的基点		
	主点 $\beta=1$	焦点	节点 $\gamma=1$
单球面	顶点	物方焦点 & 像方焦点	球心
薄透镜	光心		光心
理想光具组	物方主点&像方主点		$x_J=-f'$ $x_J'=-f$

3° 理想光具组



光焦度(单位为屈光度D)

1° $\Phi\equiv\dfrac{n'}{s'}+\dfrac{n}{s}=\dfrac{n'-n}{r}=\dfrac{n'}{f'}=\dfrac{n}{f}$

2° $\Phi=\Phi_1+\Phi_2$

$\Phi>0$, 会聚(正透镜)

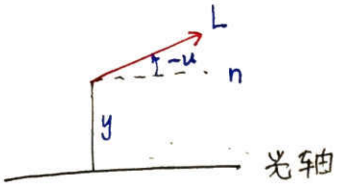
$\Phi<0$, 发散(负透镜)

$\Phi=0$,

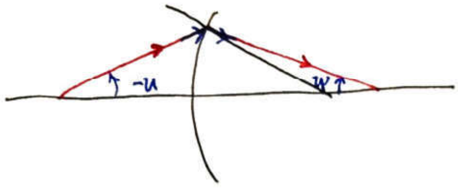
4° 共轴球面系统组合

$$\begin{cases} f=-\dfrac{f_1f_2}{\Delta} \\ f'=-\dfrac{f_1'f_2'}{\Delta} \end{cases}$$
, Δ为光学间距, $\Delta=d-f_1'-f_2$

$\Phi=\Phi_1+\Phi_2-\dfrac{d}{n}\Phi_1\Phi_2$, n为L_1L_2之间的n.

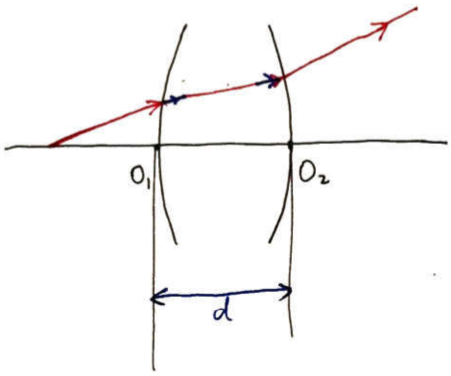


$$L=\begin{bmatrix} nu \\ y \end{bmatrix}$$



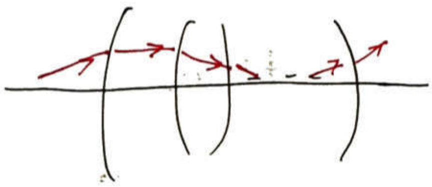
$$L'=RL \quad ,其中\ R=\begin{bmatrix} 1 & \Phi \\ 0 & 1 \end{bmatrix}$$

折射矩阵



$$L_2=T_{21}L_1' \quad ,其中\ T_{21}=\begin{bmatrix} 1 & 0 \\ -\frac{d}{n_1'} & 1 \end{bmatrix}$$

平移矩阵



$$L_N'=R_N T_{N,N-1} R_{N-1} T \cdots R_3 T_{32} R_2 T_{21} R_1 L_1$$

$$S=R_N T_{N,N-1}\cdots R_1$$

系统矩阵（始末均为R）

$$A=T_{B'N}\,S\,T_{1B}=\begin{bmatrix} \frac{1}{\beta} & \Phi \\ 0 & \beta \end{bmatrix}$$

物像矩阵（始末均为T）

推论：厚透镜的系统矩阵 $S=R_2T_{2,1}R_1=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ 中，

$$\begin{cases} \star\, S_{12}=\Phi=\Phi_1+\Phi_2-\frac{d}{n_1'}\Phi_1\Phi_2 \\ S_{11}=1-\frac{d}{n_1'}\Phi_2 \\ S_{22}=1-\frac{d}{n_1'}\Phi_1 \\ S_{21}=-\frac{d}{n_1'} \\ |S|=1 \end{cases}$$

顶主距 $\begin{cases} l_H=\frac{n_1(S_{11}-1)}{S_{12}}=\frac{-n_1\cdot d\Phi_2}{n_1'\cdot\Phi} \\ l_{H'}'=\frac{n_2'(S_{22}-1)}{S_{12}}=\frac{-n_2'\cdot d\Phi_1}{n_1\cdot\Phi} \end{cases}$

顶焦距 $\begin{cases} l_F=\frac{n_1\cdot S_{11}}{S_{12}} \\ l_{F'}'=\frac{n_2'\cdot S_{22}}{S_{12}} \end{cases}$

光波中 电场与磁场有 $\sqrt{\varepsilon_0\varepsilon_r}\,E=\sqrt{\mu_0\mu_r}\,H$

光波的传播速度为 $V=\dfrac{c}{\sqrt{\varepsilon_r\mu_r}}$，其中~~差字~~真空中光速 $c=\dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$.

光学介质的折射率 $\boxed{n=\sqrt{\varepsilon_r\mu_r}}\ \overset{\text{非铁磁性}}{\approx}\ \sqrt{\varepsilon_r}$

单色平面光波的波函数 $E=E_0\cos\left[\omega\left(t-\frac{x}{v}\right)-\varphi_0\right]$

$=E_0\cos(\omega t-kx-\varphi_0)$

其中圆波数 $k=\frac{\omega}{v}=\frac{2\pi}{\lambda'}=\frac{2\pi n}{\lambda}$

光的能流密度 S（坡印廷矢量）$\vec{S}=\vec{E}\times\vec{H}$ 对时间的平均值为光强.

$$S=EH=\sqrt{\frac{\varepsilon_0\varepsilon_r}{\mu_0\mu_r}}\,E^2=c\,\varepsilon_0 n\,E^2$$

$$I=\frac{1}{2}c\,\varepsilon_0 n\,E_0^2$$

相干光的必要条件：① 叠加光波有互相平行的光振动分量

② 波长 & 频率相等 ×

③ 光波在叠加区相位差恒定

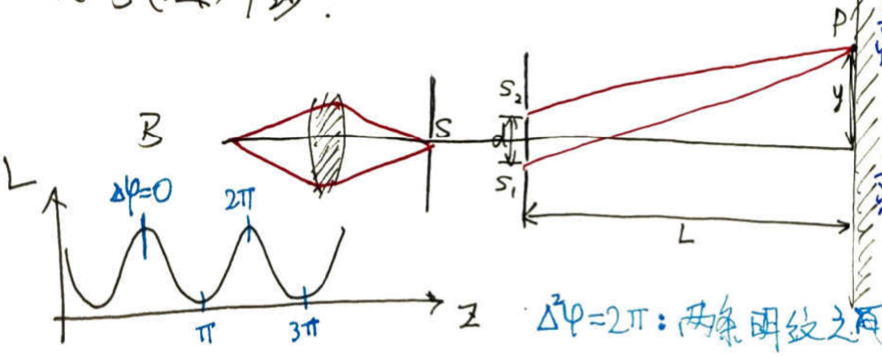
$\boxed{I=I_1+I_2+2\sqrt{I_1I_2}\cos\delta\text{，其中}\delta\text{为相位差}}$.

$$\left(\begin{aligned}I_{max}&=(\sqrt{I_1}+\sqrt{I_2})^2\\ I_{min}&=(\sqrt{I_1}-\sqrt{I_2})^2\end{aligned}\right).$$

条纹可见度 $V=\dfrac{2\sqrt{I_1I_2}}{I_1+I_2}$ $\to 1$ 最明显 / $\to 0$ 不可见

☆ $\boxed{\left(V\equiv\dfrac{I_{max}-I_{min}}{I_{max}+I_{min}}\right)}$

双孔（缝）干涉.

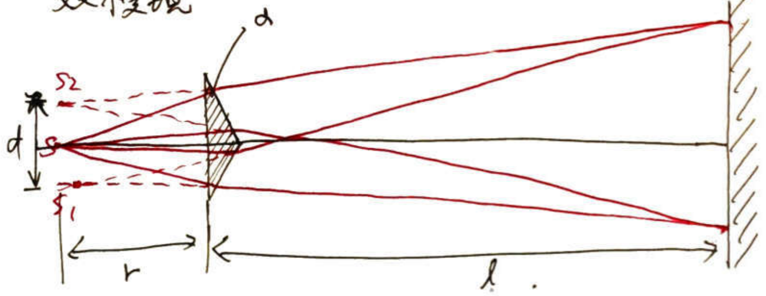


亮条纹位置 $y_b=\frac{L}{d}k\lambda$，$k=0,\pm1,\pm2,\dots$

亮条纹间距 $e=\frac{L}{d}\lambda$

$\Delta\varphi=2\pi$：两条明纹之间

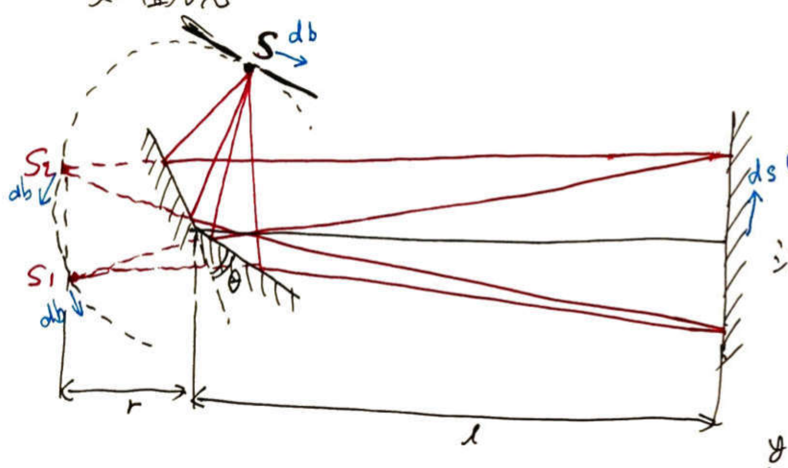
双棱镜



亮条纹间距

$$e=\frac{r+l}{d}\lambda=\frac{(r+l)\lambda}{2r(n-1)\alpha}$$

双面镜

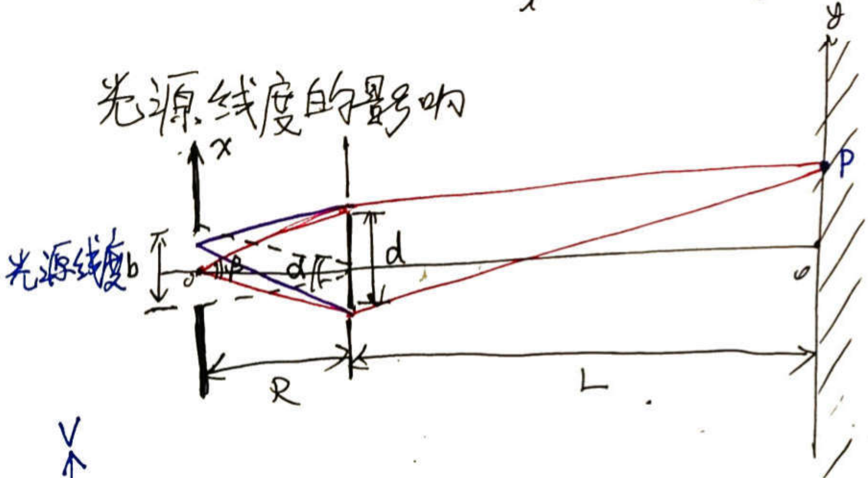


$$e=\frac{\lambda(r+l)}{2\sin\theta\cdot r}$$

注：反射光波相对入射光波有相位跃变π

$$\frac{ds}{db}=\frac{l}{r}$$

光源线度的影响

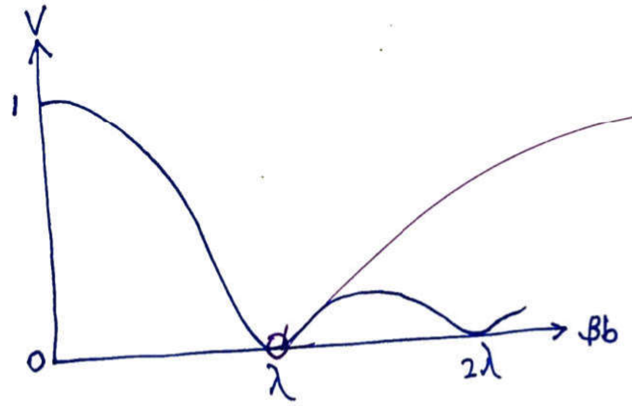


干涉孔径角 $\beta\approx\frac{d}{R}$.

P点干涉光强为

$$I_p=2I_1\left(1+\frac{\sin\frac{\pi}{\lambda}\beta b}{\frac{\pi}{\lambda}\beta b}\cos\delta_0\right)$$

$$V=\frac{\left|\sin\frac{\pi}{\lambda}\beta b\right|}{\frac{\pi}{\lambda}\beta b}$$



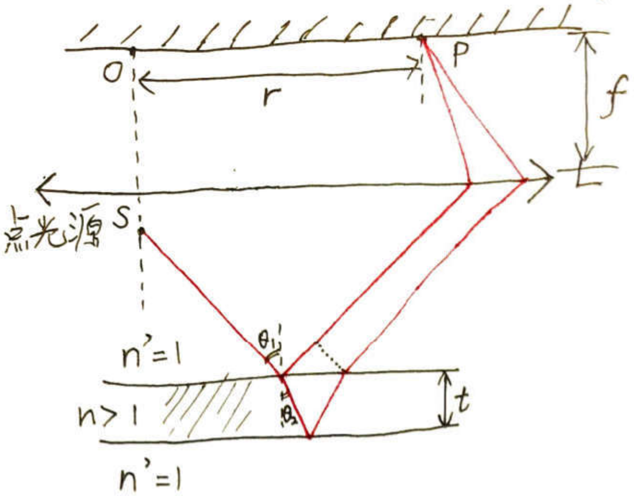
临界宽度（对应第一个V的零点）

$$b_c=\frac{\lambda}{\beta}=\frac{R\lambda}{d}$$

注：可用于测量红巨星的视直径.

等倾条纹（定域）

注：~~反~~射只有在光疏介质一方才有半波损失.

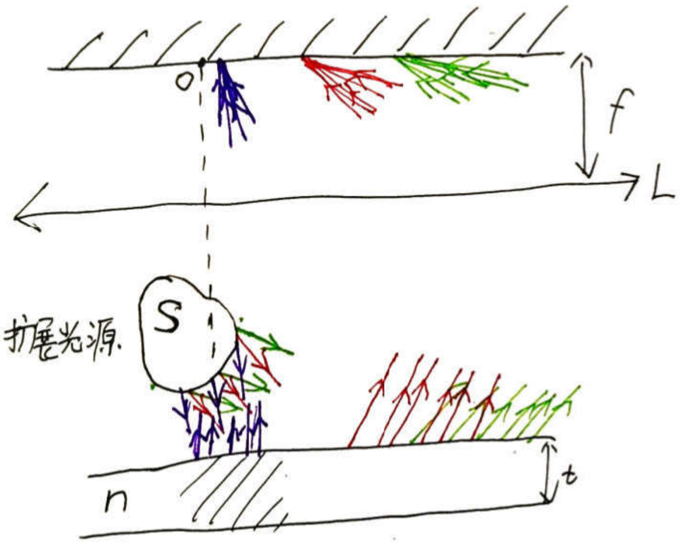


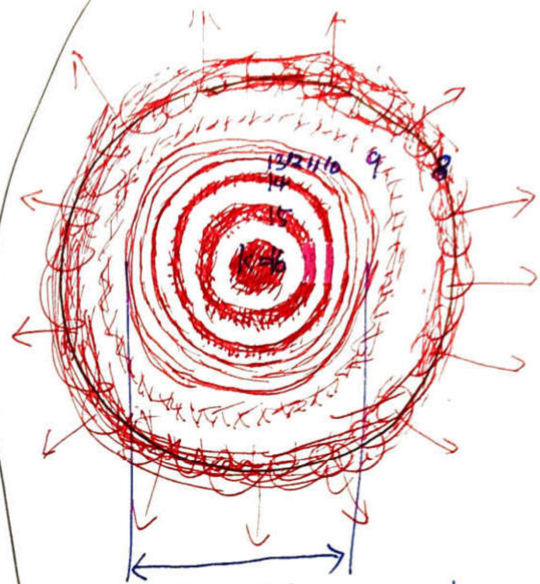
光程差 $\Delta L = 2nt\cdot\cos\theta_2 + \frac{\lambda}{2}$（严格）

k级亮纹：$\Delta L = k\lambda$；

$\cos i_k = \frac{(k-\frac{1}{2})\lambda}{2nt}$；$(i_k \equiv \theta_2)$.

亮纹间距：$\Delta r \approx \frac{-\lambda}{2nt\cdot\sin i_k}$ (

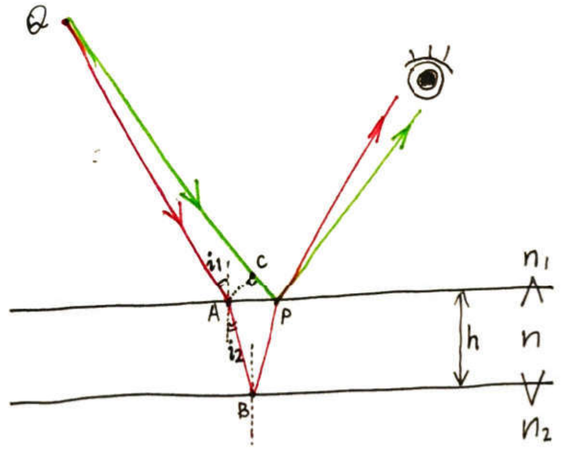




每冒出一个环 $\Leftrightarrow t \mathrel{+}= \frac{\lambda}{2n}$

应用：增透镜：镀膜厚度为 $\frac{\lambda}{4n}$ 或其奇数倍数.

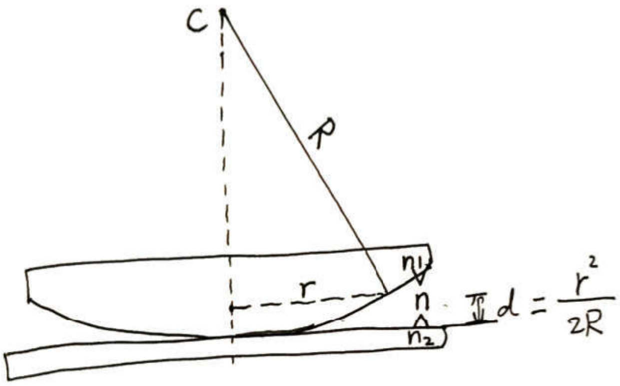
等厚干涉（定域）



$\Delta L(P) \approx 2nh\cos i_2$

$\Longrightarrow$ 若Q无穷远／平行光源，
则干涉条纹（明、暗）为等厚线，
其厚度为 $\begin{cases} h=\dfrac{2k\cdot\lambda}{4n\cdot\cos i_2} & (明) \\ h=\dfrac{(2k+1)\cdot\lambda}{4n\cdot\cos i_2} & (暗) \end{cases}$

光线垂直入射时，每相差一个条纹，$\Delta h=\dfrac{\lambda}{2n}$.

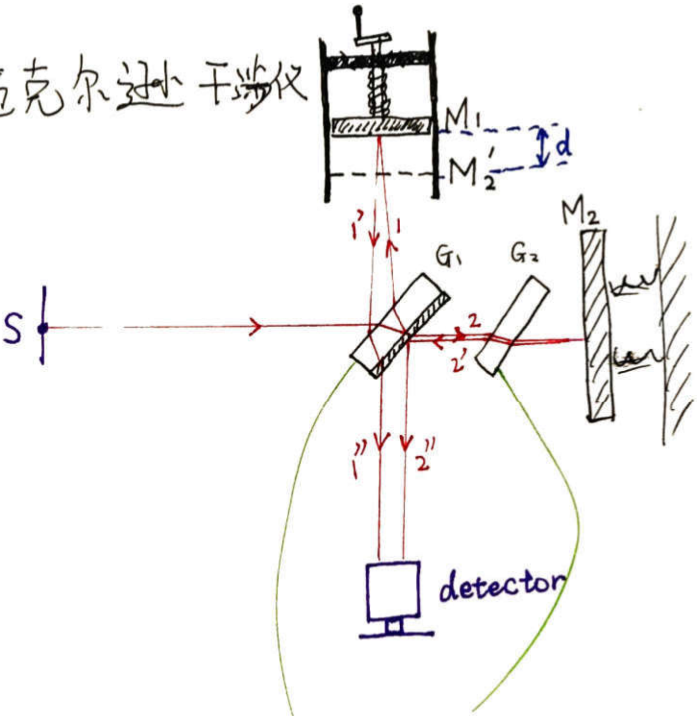


$\Delta L = 2d + \dfrac{\lambda}{2}$

暗 $r_d = \sqrt{mR\lambda}$ ，$m=0,1,2,\cdots$

明 $r_b = \sqrt{(m-\frac{1}{2})R\lambda}$ ，$m=1,2,\cdots$

迈克尔逊干涉仪



S	detector
激光	屏
扩展光源	眼

条纹移动N条：$\Delta d = N\cdot\dfrac{\lambda}{2}$

10个图：教书P98.

若 G_1&G_2 材质相同，则 G_1&G_2 等厚且等倾角（确保 M_2' 的位置不随波长而变）使0级暗纹在中心重合

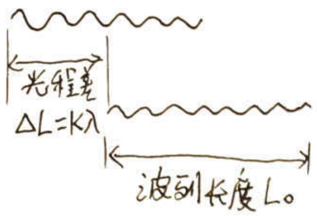
分光片 补偿片 — common-mode noise rejection

玻璃，光程差随温度变化等

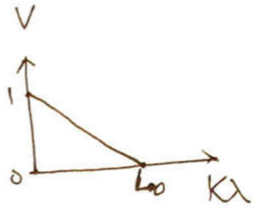
热膨胀 收缩

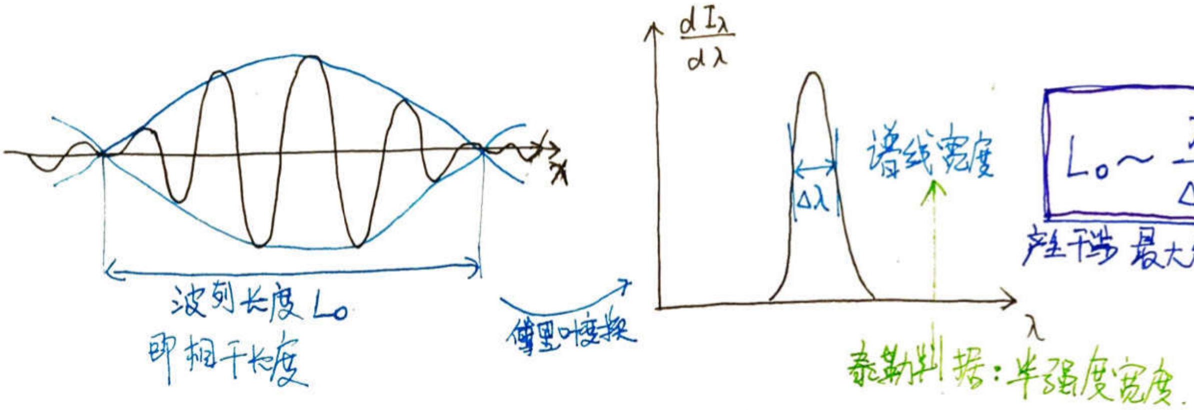
折射率变化

时间相干性



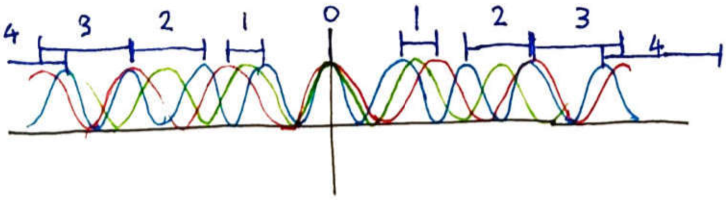
$$V = \begin{cases} 1 - \dfrac{k\lambda}{L_0} & (k\lambda \le L_0 \text{时}) \\ 0 & (k\lambda > L_0 \text{时}) \end{cases}$$





$$\boxed{L_0 \sim \frac{\lambda^2}{\Delta\lambda}}$$

产生干涉最大允许的光程差



干涉条纹消失：

$$\lambda(m+1) = m(\lambda + \Delta\lambda)$$

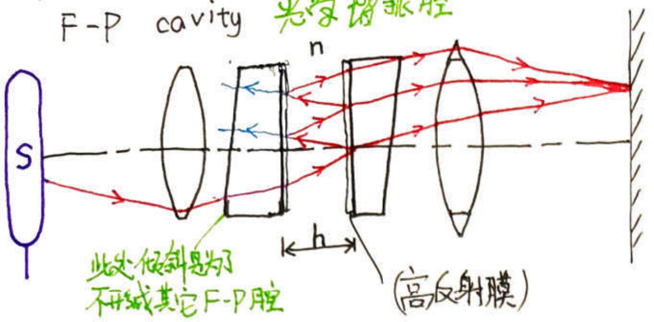
$$\Rightarrow \boxed{m = \frac{\lambda}{\Delta\lambda}}$$

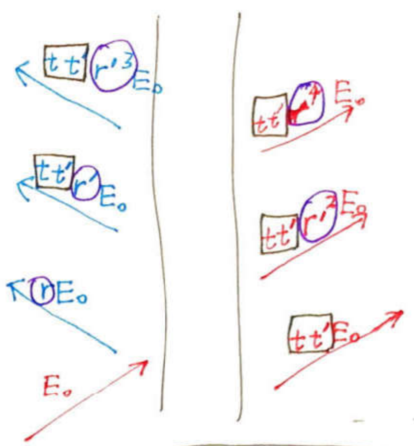
能分辨的最高级次

分振幅法中的减幅多光束干涉

法布里-珀罗干涉仪（非定域）

F-P cavity 光学谐振腔



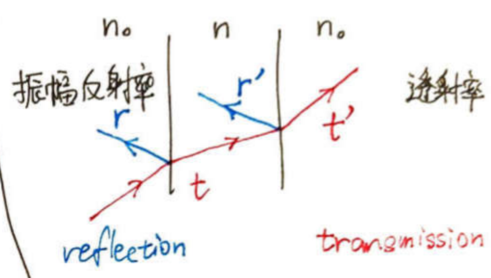


叠加后：$\delta = \frac{4\pi}{\lambda} nh\cdot\cos\theta$

合反射光复振幅 $E_R = \frac{r(1-e^{i\delta})}{1-r^2e^{i\delta}}E_0$

合透射光复振幅 $E_T = \frac{(1-r^2)}{1-r^2e^{i\delta}}E_0$

斯托克斯关系 $\begin{cases} r=-r' \\ r^2+tt'=1 \end{cases}$

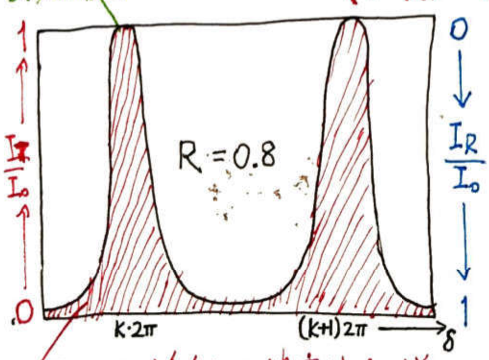


光强反射率 $R \equiv r^2$

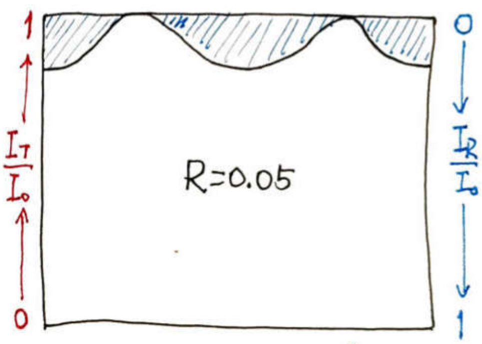
$$I_R = \frac{4R\sin^2\frac{\delta}{2}}{(1-R)^2+4R\sin^2\frac{\delta}{2}} I_0$$

$r\to1$ 时非常尖锐，即 I_T 对 $\delta=\frac{4\pi nh}{\lambda}\cos\theta$ 非常敏感，可探测引力波

$$I_T = \frac{(1-R)^2}{(1-R)^2+4R\sin^2\frac{\delta}{2}} I_0$$



R大时，透射干涉亮纹细锐



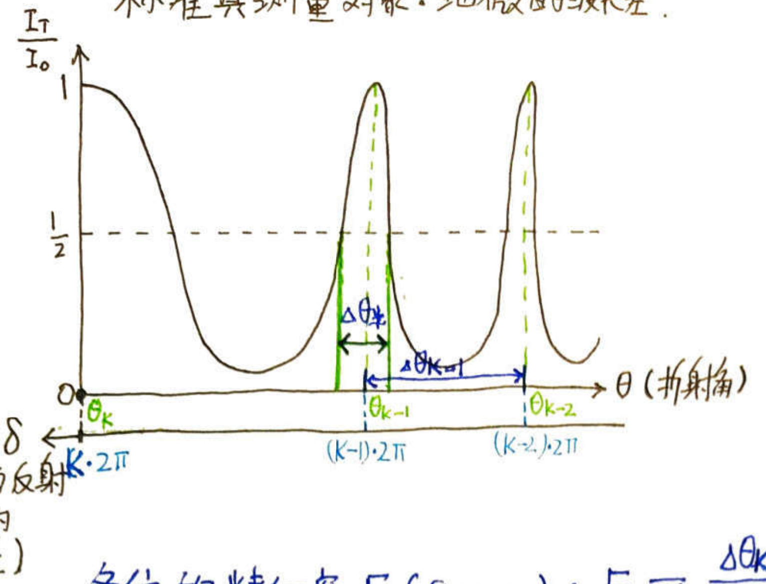
R小时，$I_R \approx 4R\sin^2\frac{\delta}{2}\cdot I_0$

等幅双光束干涉光强

~~Cof~~ Coefficient of finesse 精细度/锐度系数 $F=\left(\frac{2r}{1-r^2}\right)^2$ 也可看作光在F-P腔中反射的次数

Airy function $\frac{I_t}{I_0} = \frac{1}{1+F\sin^2\frac{\delta}{2}}$

标准具测量对象：细微的波长差.



同等倾干涉：$\delta = \frac{4\pi}{\lambda} nh\cos\theta$

⇒ 半强度宽度

$$\Delta\theta_半 = \frac{\lambda}{4\pi nh\sin\theta_k}\cdot\Delta\delta_半$$

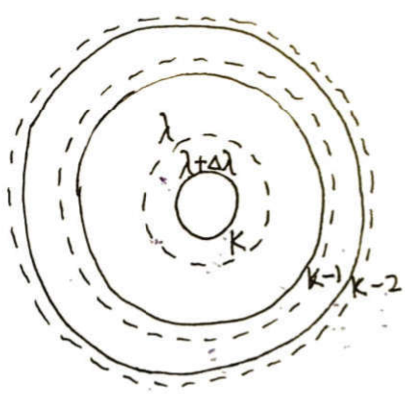
$$\Delta\delta_半 = \frac{2(1-R)}{\sqrt{R}}$$

条纹的精细度 F (fineness)：$F \equiv \frac{\Delta\theta_K}{\Delta\theta_半} = \frac{\pi\sqrt{R}}{1-R}$ （与θ、δ无关）.
（另一个定义）

角色散 $D_\theta \equiv -\frac{d\theta}{d\lambda} = \frac{1}{\lambda\tan\theta}$ （波长大的谱线更靠近中心）

自由光谱范围 $\Delta\lambda_{fsr} \approx \frac{\lambda}{K} = \frac{\lambda^2}{2nh}$
（标准具的量程） $\Delta f_{fsr} \approx \frac{c}{2nh}$
free spectral range

$$\boxed{\begin{aligned}&\theta(\lambda, K) = \theta(\lambda+\Delta\lambda, K-1)\\ &\Rightarrow \underline{K\lambda} = \underline{2nh\cos\theta} = \underline{(K-1)(\lambda+\Delta\lambda)}\end{aligned}}$$



色分辨本领的泰勒判据

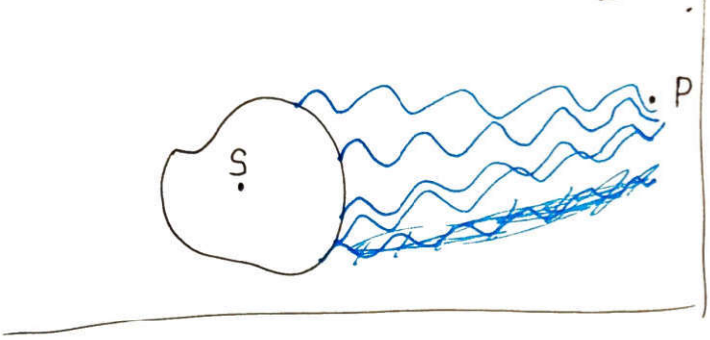
$\Delta\theta < \Delta\theta_半$，不可分辨；
$\Delta\theta = \Delta\theta_半$，恰好能分辨；
$\Delta\theta > \Delta\theta_半$，可分辨

⇒ 最小可辨波长差

$$\delta\lambda = \frac{\lambda}{KF} = \frac{\lambda}{K}\cdot\frac{1-R}{\pi\sqrt{R}} = \frac{\lambda^2}{2nh}\cdot\frac{1-R}{\pi\sqrt{R}}$$

色分辨本领 $A \equiv \frac{\lambda}{\delta\lambda} = KF = K\frac{\pi\sqrt{R}}{1-R} = \frac{2nh}{\lambda}\cdot\frac{\pi\sqrt{R}}{1-R}$ （与θ、δ无关）

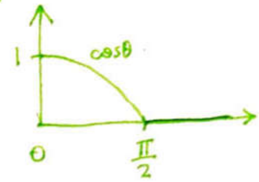
惠更斯-菲涅耳原理 ：次波振幅叠加



菲涅耳衍射积分公式

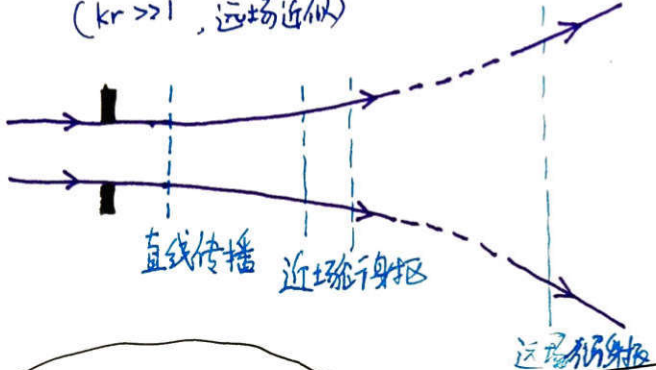
$$\tilde{E}(P)=C\cdot\iint_{\Sigma} F(\theta)\cdot E(\theta)\cdot\frac{e^{ikr}}{r}\,d\sigma$$

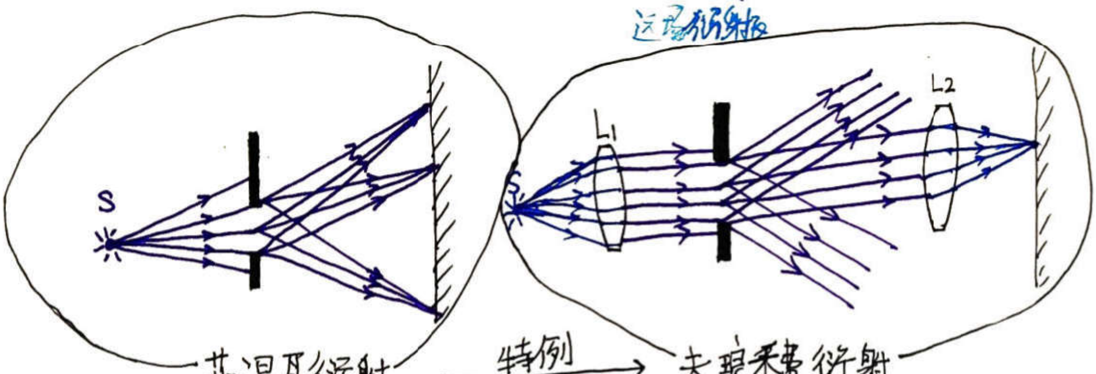
倾斜因子



菲涅耳-基尔霍夫衍射公式 ： $F(\theta)=\frac{\cos\theta_0+\cos\theta}{2}$ ， $C=\frac{1}{i\lambda}$

（$kr\gg1$，远场近似）

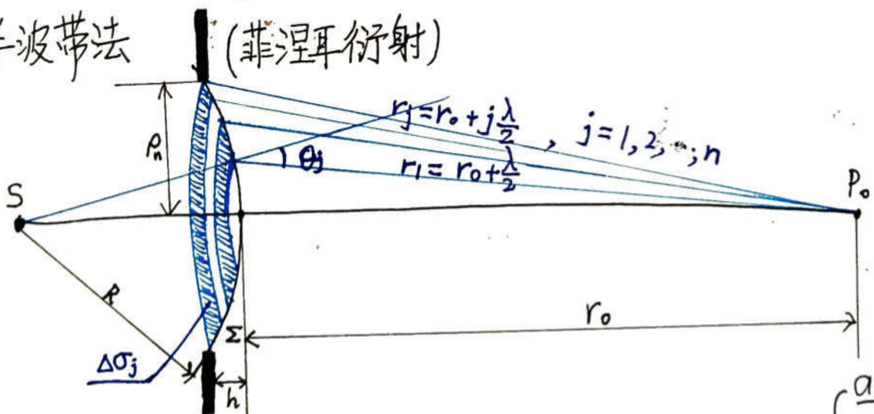




菲涅耳衍射 $\xrightarrow{\text{特例}}$ 夫琅禾费衍射

（近场衍射） （远场衍射）

半波带法 （菲涅耳衍射）



$$E(P_0)=\sum_{j=1}^{n}E_j(P_0)=\sum_{j=1}^{n}(-1)^{j+1}\cdot\underbrace{|E_j(P_0)|}_{a_j}=\begin{cases}\frac{a_1+a_n}{2}, & n\text{奇}\\ \frac{a_1-a_n}{2}, & n\text{偶}\end{cases}\overset{n\text{很小}}{=\!=}\begin{cases}a_1, & n\text{奇}\\ 0, & n\text{偶}\end{cases}$$

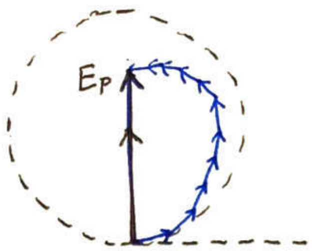
$$a_j\propto(1+\cos\bar{\theta}_j)\cdot\frac{\Delta\sigma_j}{r_j}\propto\frac{\pi R\lambda}{R+r_0}(1+\cos\bar{\theta}_j),$$

$$\Delta\sigma_j=\frac{\pi R\lambda}{R+r_0}\cancel{\left(r_0+j\frac{\lambda}{2}-\frac{\lambda}{4}\right)}$$

$$\bar{r}_j=\cancel{r_0+j\frac{\lambda}{2}-\frac{\lambda}{4}}$$

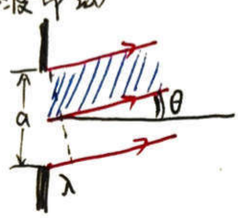
$$\boxed{n=\frac{\rho_n^2(R+r_0)}{\lambda r_0 R}}$$

矢量图解法



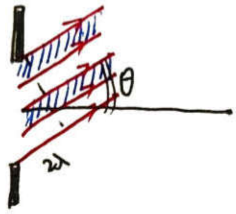
单缝夫琅禾费衍射

(半波带法)



$K=1$

1级暗纹

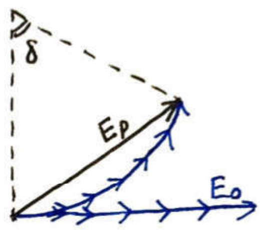


$K=2$

$a\sin\theta = K\lambda$ ，$K=0$ 时：中心明纹（稍偏）

$K=\pm1,\pm2,\cdots$ 明暗纹（准确）

(矢量图解法)



$I_p \propto E_p^2 \propto \left(\frac{\sin\alpha}{\alpha}\right)^2$，其中 $\alpha = \frac{\delta}{2} = \frac{\pi a\sin\theta}{\lambda}$

$I_p = I_0 \operatorname{sinc}^2\alpha$

(积分法)

比例因子 $K = \frac{1}{i\lambda} = \frac{1}{\lambda}e^{-i\frac{\pi}{2}}$

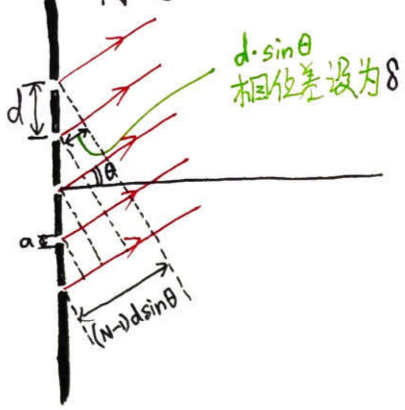
$$E_\theta = C_1\cdot\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-ik\Delta r}\,dx = C_1\cdot\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-ik\sin\theta\cdot x}\,dx = C_2\cdot\frac{\sin\left(\frac{ka\pi\sin\theta}{2}\right)}{\frac{ka\pi\sin\theta}{2}}$$

$$\Rightarrow I_\theta \propto \operatorname{sinc}^2\left(\frac{k\pi a\sin\theta}{2}\right) = \operatorname{sinc}^2\left(\frac{a\pi\sin\theta}{2\lambda}\right)$$

求导得 极强位置为：$\sin\theta = 0,\ \pm1.43\frac{\lambda}{a},\ \pm2.46\frac{\lambda}{a},\ \pm3.47\frac{\lambda}{a},\ \pm4.48\frac{\lambda}{a}$.

多缝的夫琅禾费衍射

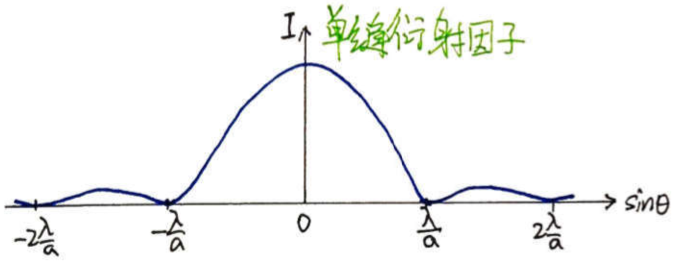
N=5 时

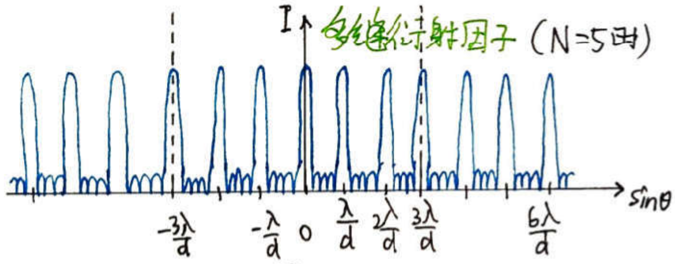


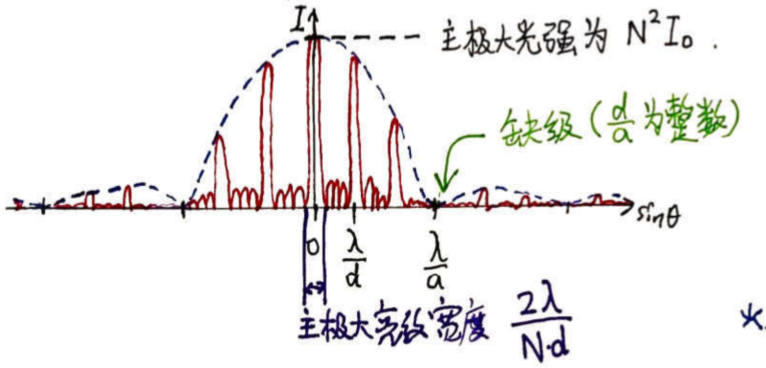
$$A_{N\theta} = \sum_{j=0}^{N-1} A_{1\theta} e^{i\cdot j\delta} = A_{1\theta} \frac{\sin\frac{N}{2}\delta}{\sin\frac{1}{2}\delta} e^{i(N-1)\delta/2}$$ (等比数列).

$$A_{1\theta} = A_{10}\cdot \mathrm{sinc}\left(\frac{a\pi\cdot\sin\theta}{2\lambda}\right)$$

$$I_{N\theta} = I_{10}\cdot \underbrace{\boxed{\mathrm{sinc}^2\left(\frac{a\pi\cdot\sin\theta}{2\lambda}\right)}}_{\text{单缝衍射因子}} \cdot \underbrace{\boxed{\left(\frac{\sin\frac{Nd\pi\cdot\sin\theta}{2\lambda}}{\sin\frac{d\cdot\pi\cdot\sin\theta}{2\lambda}}\right)^2}}_{\text{多缝衍射因子}}$$



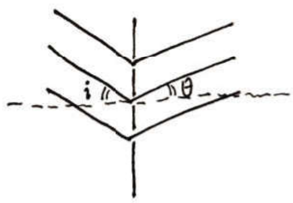




透射光栅.

☆光栅方程: $d\cdot\sin\theta = K\lambda$. (K为亮纹级数)

斜入射光栅方程:



$$d\cdot(\sin i + \sin\theta) = K\lambda.$$

注: 可调整中心亮纹的方向、位置

角色散 $D_\theta \equiv \frac{d\theta}{d\lambda} = \frac{K}{d\cdot\cos\theta}$

线色散 $D_l \equiv \frac{dl}{d\lambda} = f' D_\theta$

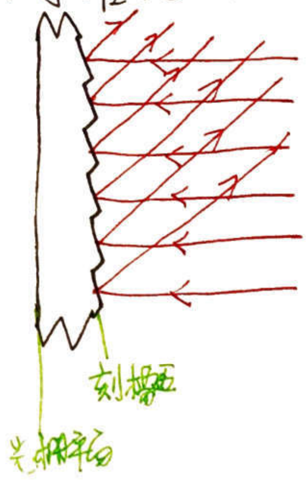
*光栅的色分辨本领 (瑞利判据)

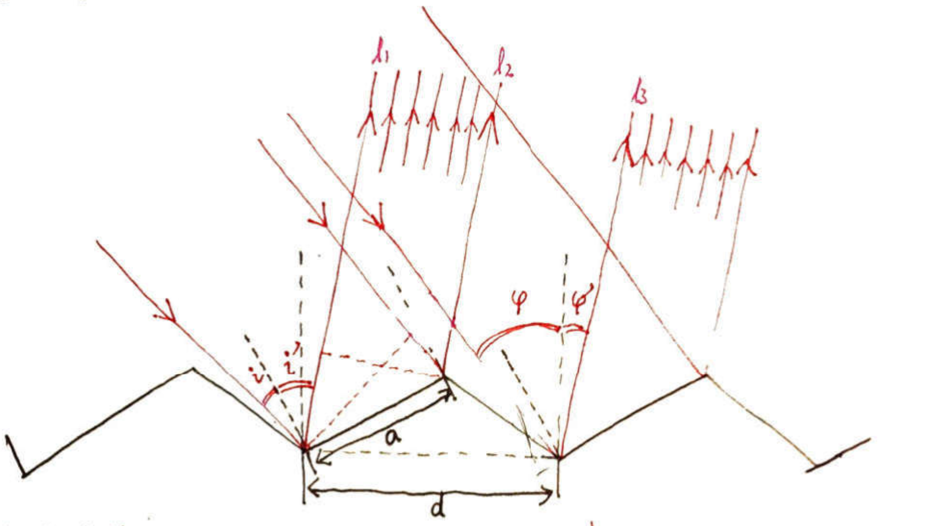
$$A \equiv \frac{\lambda}{\delta\lambda} = \underbrace{K}_{\text{光谱的级数}}\cdot \underbrace{N}_{\text{缝数}}$$

*自由光谱范围

$$\Delta\lambda = \frac{\lambda}{K}$$

闪耀光栅（反射定向光栅）

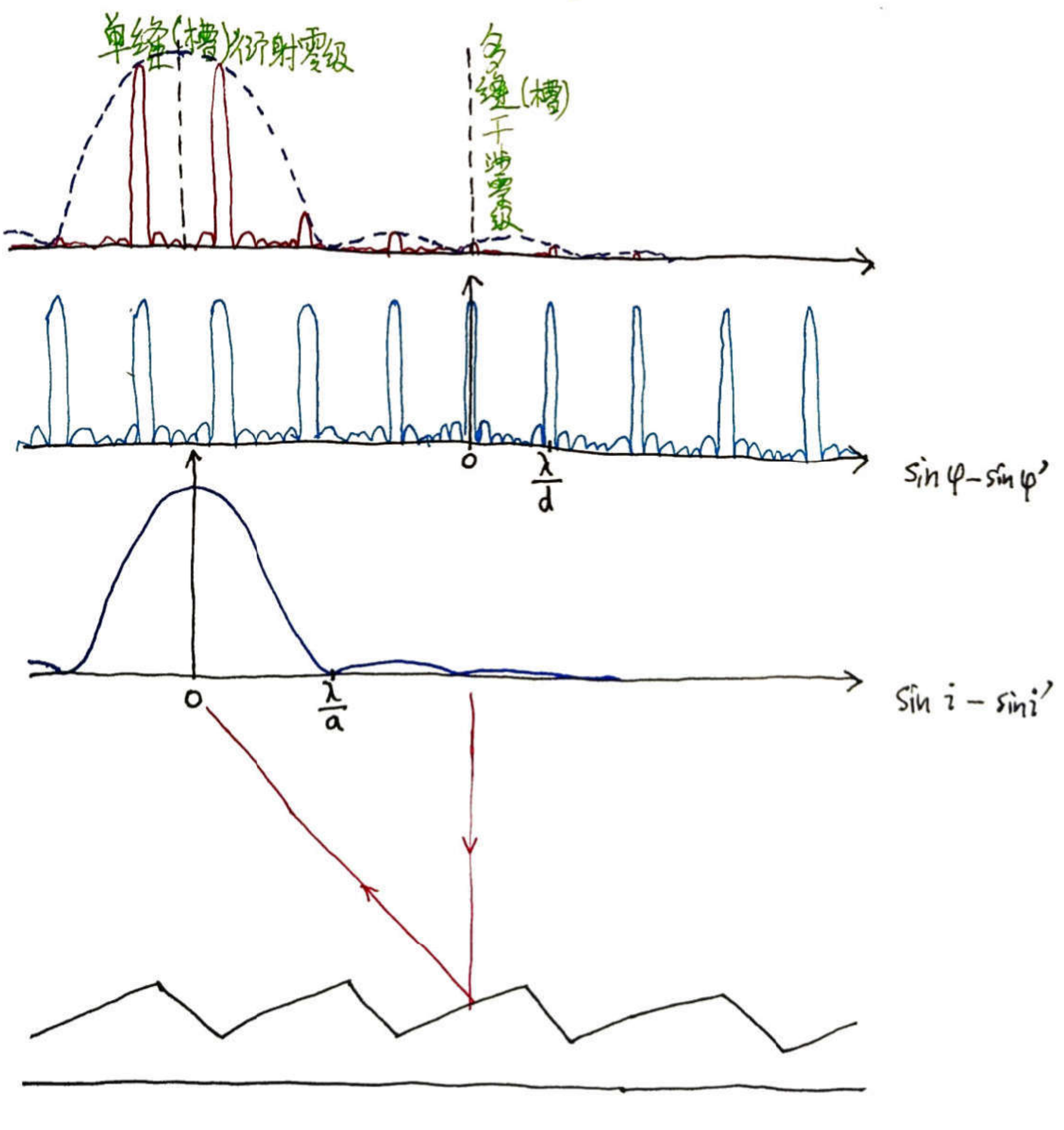




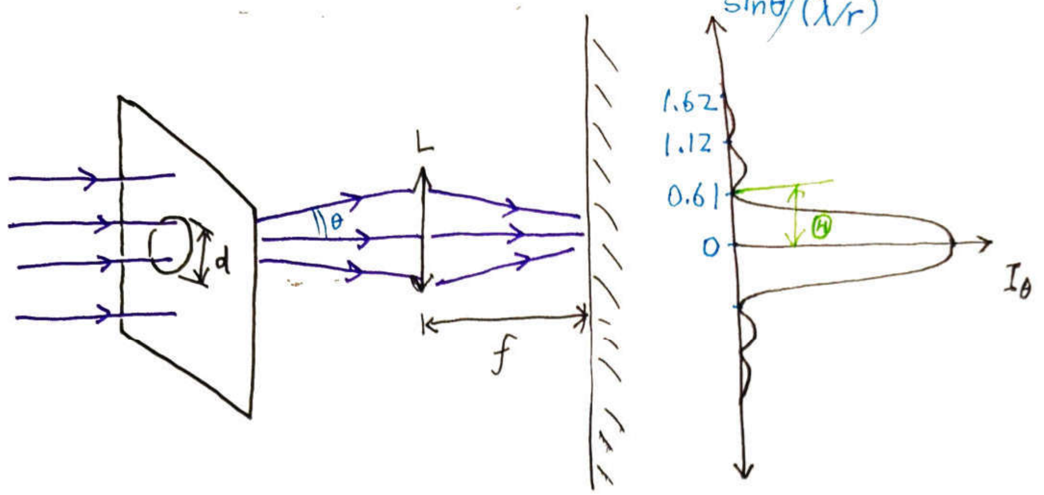
l_1与l_2光程差 $\Delta = a(\sin i - \sin i')$　相位差 $\delta = \frac{2\pi a}{\lambda}(\sin i - \sin i')$

l_1与l_3光程差 $\Delta' = d(\sin\varphi - \sin\varphi')$　相位差 $\delta' = \frac{2\pi d}{\lambda}(\sin\varphi - \sin\varphi')$

光强分布
$$I = I_0 \underbrace{\boxed{\left(\frac{\sin\frac{\delta}{2}}{\frac{\delta}{2}}\right)^2}}_{\text{单槽衍射因子}} \underbrace{\boxed{\left(\frac{\sin N\frac{\delta'}{2}}{\frac{\delta'}{2}}\right)^2}}_{\text{缝间干涉因子}}.$$

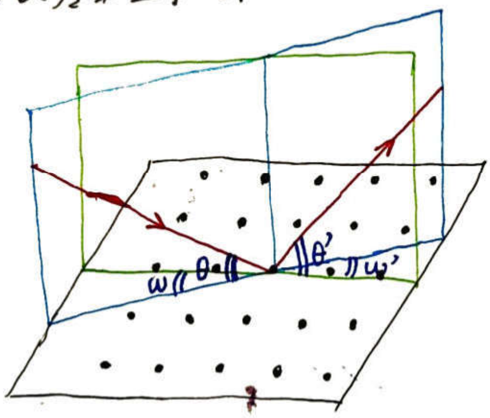


圆孔夫琅禾费衍射



中心亮斑(艾里斑)角半径 $\Theta = \dfrac{0.61\lambda}{r} = \dfrac{1.22\lambda}{d}$

单面晶格的零级主极强



零级主极强的条件:(镜面反射线方向)

① $\theta' = \theta$

衍射角　　掠射角

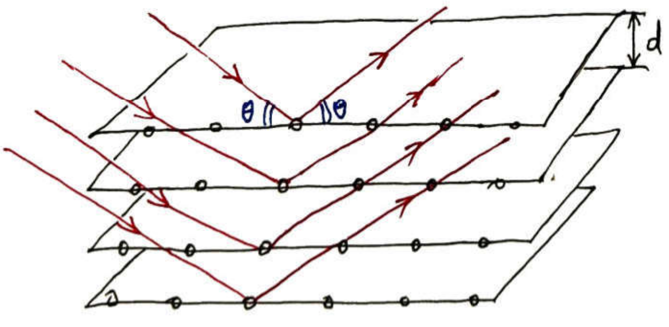
② $\omega' = \omega$.

面间干涉的零级主极强的条件: 布拉格条件

布拉格公式:

$2d\sin\theta = k\lambda$,

其中k为任意正整数

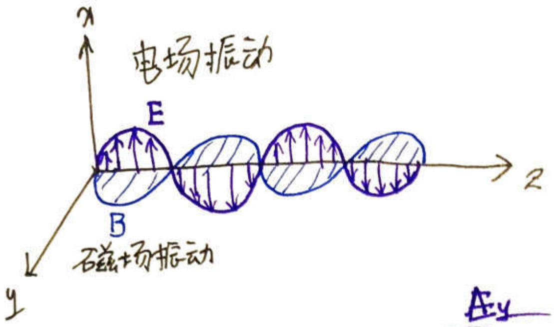


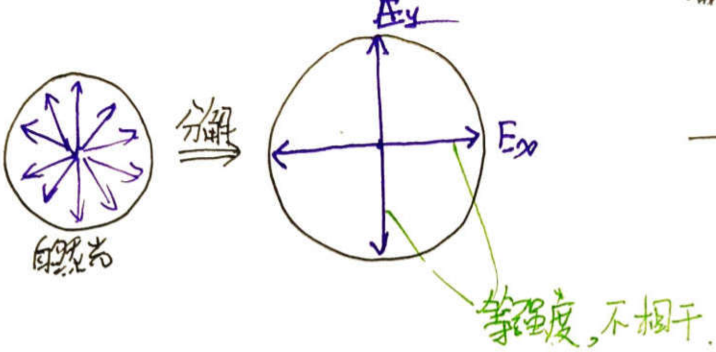
注:一个晶体内有许多晶面族

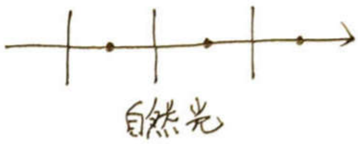
通常只考虑 $\vec{E}$ 的振动)
(通常以 $\vec{E}$ 代表光振动矢量)

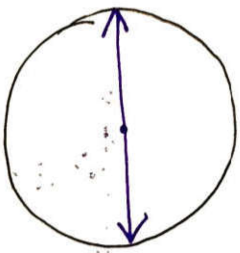
横波有偏振现象，纵波无.

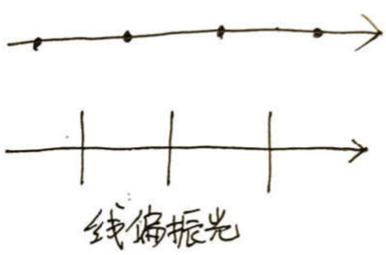
光
- 自然光
- 部分偏振光
- 偏振光
  - 线偏
  - 椭圆偏
  - 圆偏

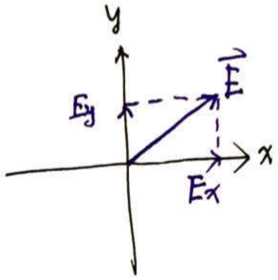












$$\begin{cases} E_x = E_{x0} \cdot \cos(\omega t) \\ E_y = E_{y0} \cdot \cos(\omega t - \varphi) \end{cases}$$

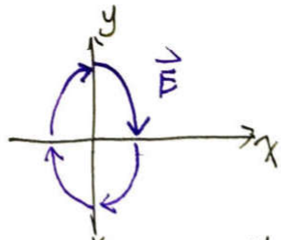
① $\varphi = k\pi$ 时，线偏振光

② $\varphi = (2k+\frac{1}{2})\pi$ 时，(顺时针) 右旋椭偏光

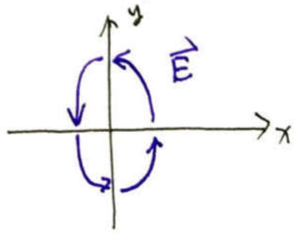
③ $\varphi = (2k-\frac{1}{2})\pi$ 时，逆　左旋椭圆偏振光

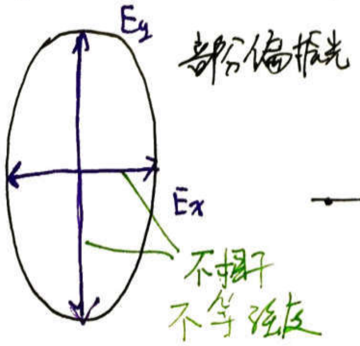
④ $\varphi =$ 其它值，椭圆偏振光.

特例：$E_{x0} = E_{y0}$ 时，椭偏光 → 圆偏振光



(迎着光来看)(接收者视角)





偏振度 $P \equiv \dfrac{I_p}{I_t} = \dfrac{I_p}{I_n + I_p}$

$\triangle$ 总光强　$\triangle$ 自然光强度　$\triangle$ 偏振光强度

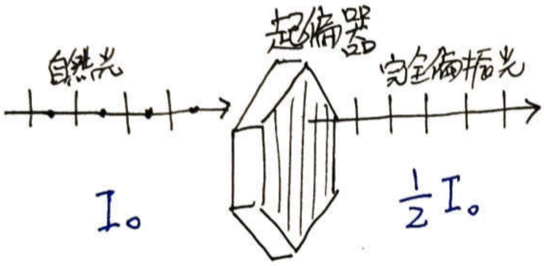
$= \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

完全偏振光 $P=1$

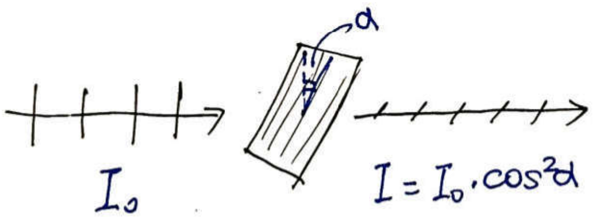
线偏振光、(椭)圆偏振光.

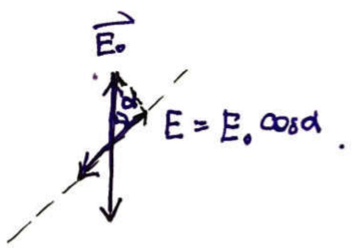
部分偏振光 $0 < P < 1$

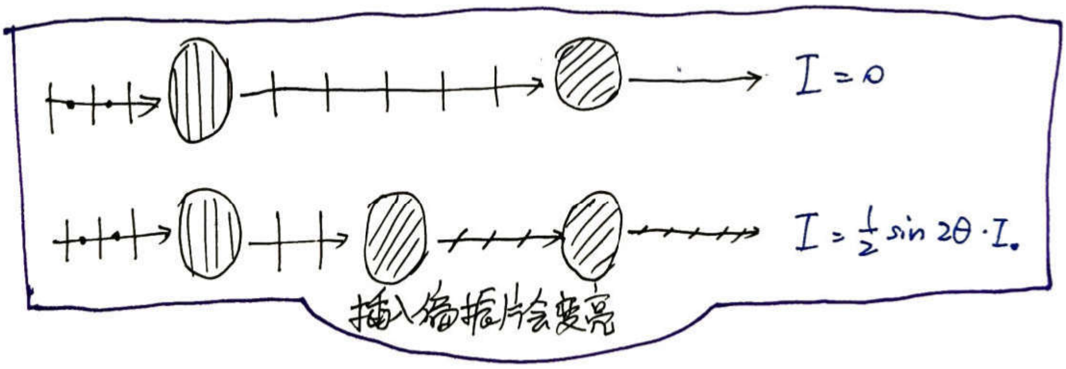
自然光 $P = 0$



马吕斯定律



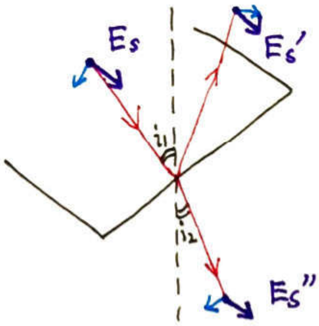




折反射中振幅的边界条件（含符号，即复振幅）

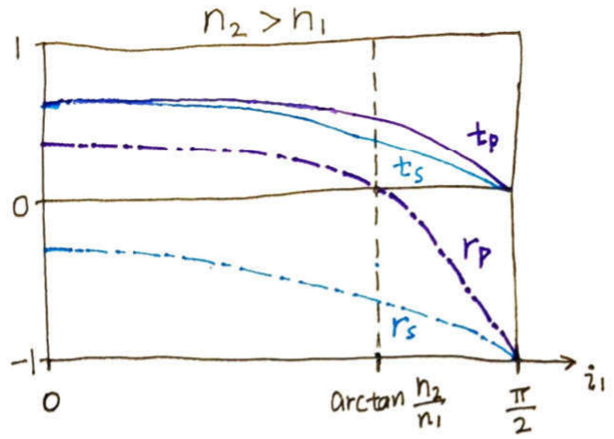
$E_{1t} = E_{2t}$ $\quad H_{1t} = H_{2t}$ （切向分量连续）

① 光矢垂直入射面

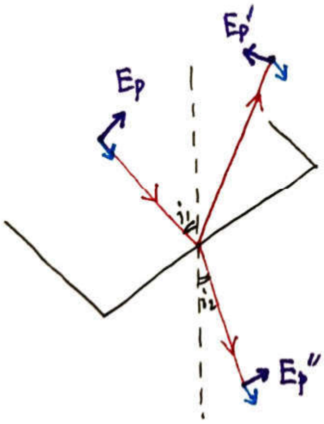


$$r_s \equiv \frac{E_{0s}'}{E_{0s}} = \frac{-\sin(i_1-i_2)}{\sin(i_1+i_2)}$$

$$t_s \equiv \frac{E_{0s}''}{E_{0s}} = \frac{2\cos i_1 \sin i_2}{\sin(i_1+i_2)}$$

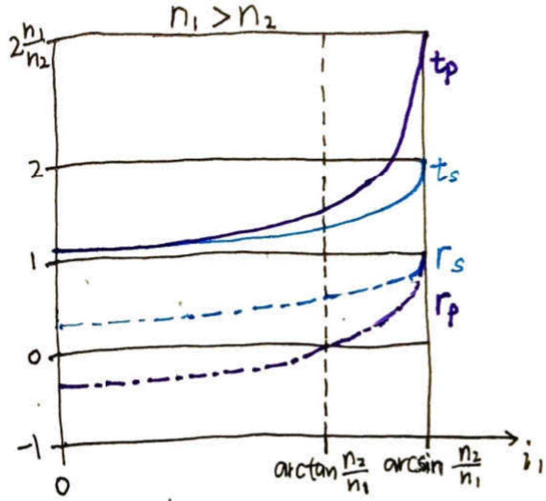


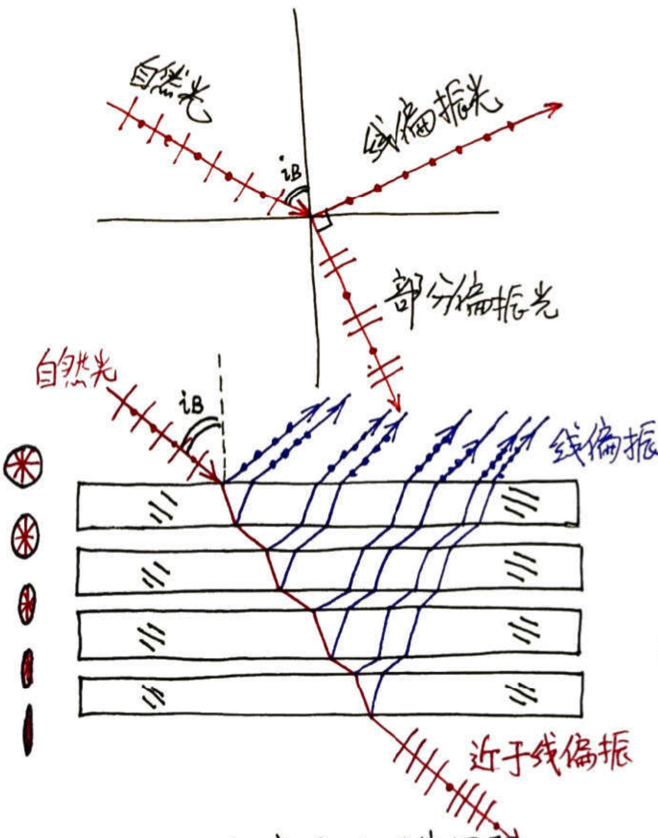
② 光矢平行入射面



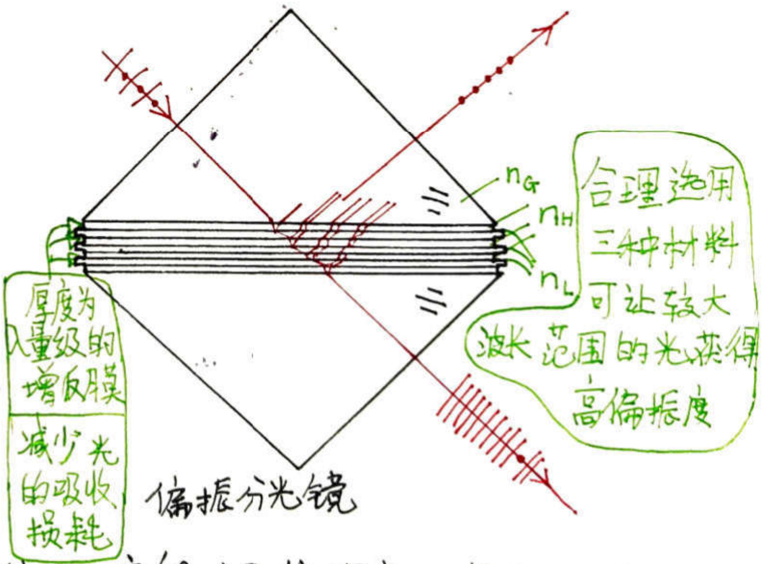
$$r_p \equiv \frac{E_{0p}'}{E_{0p}} = \frac{\tan(i_1-i_2)}{\tan(i_1+i_2)}$$

$$t_p \equiv \frac{E_{0p}''}{E_{0p}} = \frac{2\cos i_1 \sin i_2}{\sin(i_1+i_2)\cos(i_1-i_2)}$$





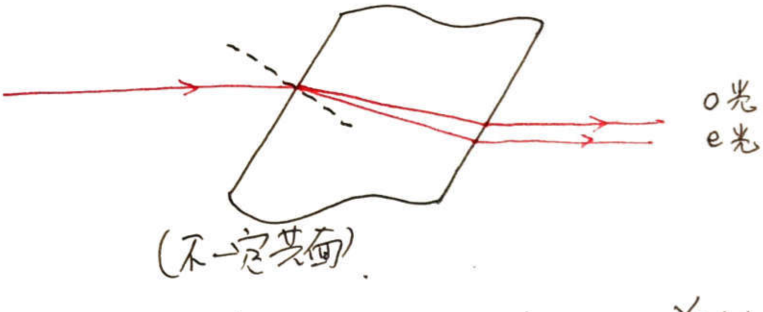
玻璃片堆（偏振器）



偏振分光镜

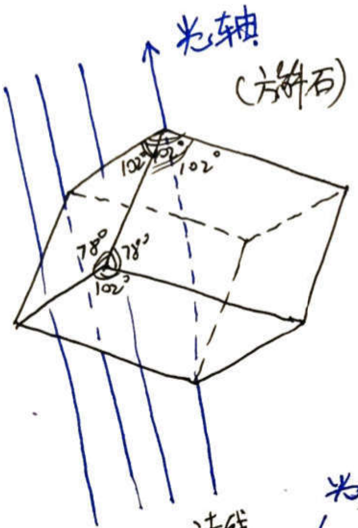
较彻底解决了偏振度与能流利用率的矛盾

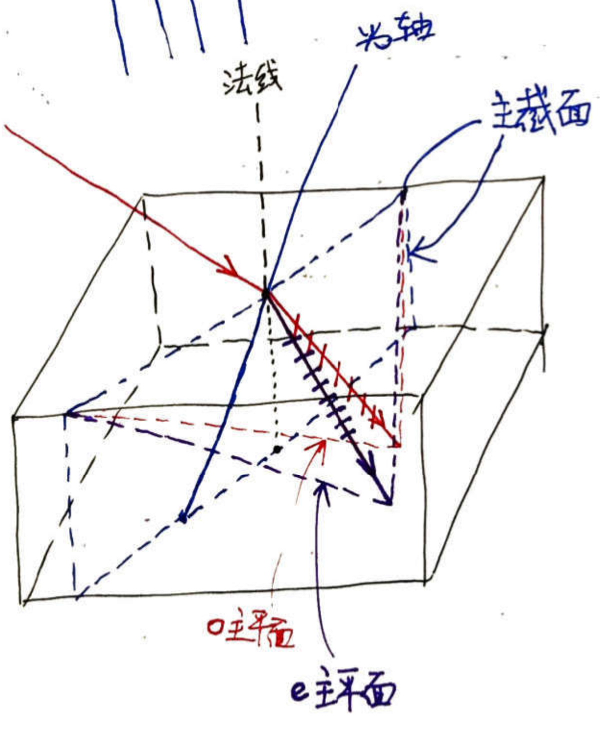
双折射



晶体的光轴

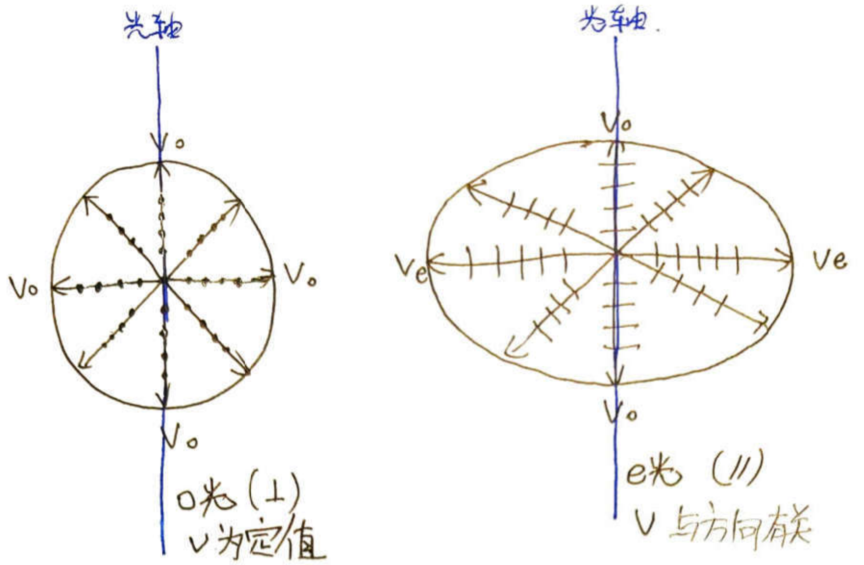
晶体 { 单轴晶体 / 双轴晶体 }

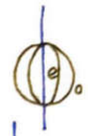


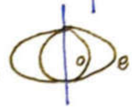


o光：振动方向与o主平面垂直

e光：振动方向与e主平面平行

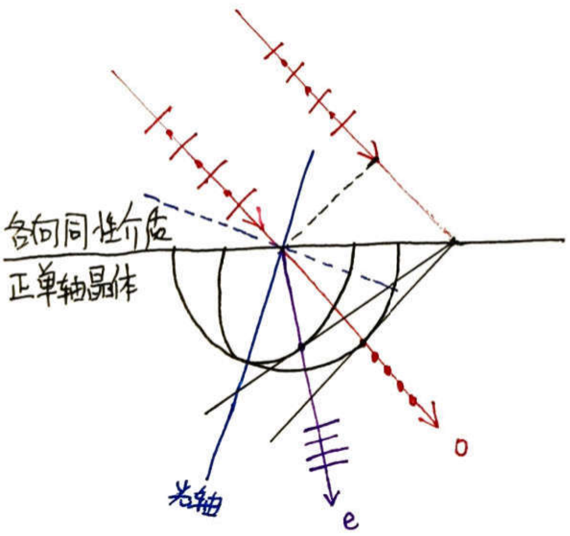


正晶体： $n_e > n_o$ ，($V_e < V_o$) e为长椭球，o在外。 

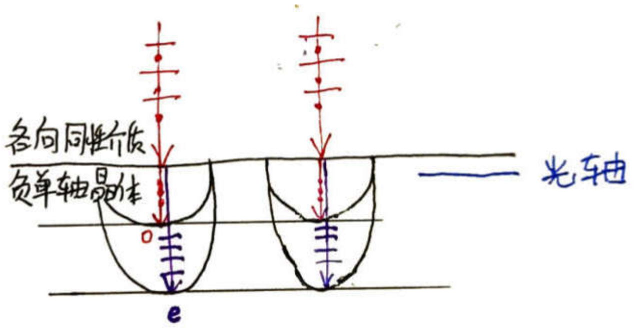
负晶体： $n_e < n_o$ ，($V_e > V_o$)，e为圆椭球，o在内。 

主折射率

惠更斯作图法



(方向不同，速度不同)

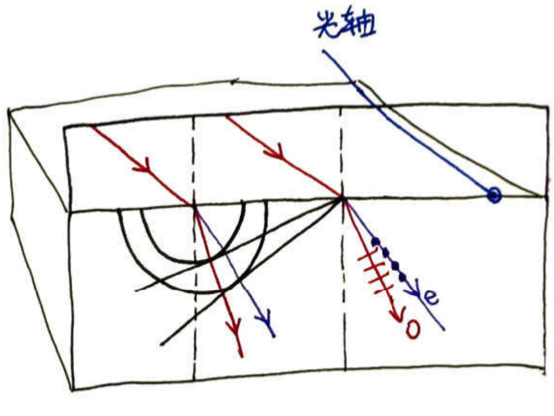


光轴平行入射面时，o圆 e椭（光轴方向）

光轴垂直入射面时，o圆 e圆（不等大）

注：此时 o光与e光都满足Snell定律

$$\begin{cases} \dfrac{\sin i}{\sin r_o} = n_o = \dfrac{c}{V_o} \\ \dfrac{\sin i}{\sin r_e} = n_e = \dfrac{c}{V_e} \end{cases}$$



光线在介质中沿光轴方向传播时，无双折射，不同于入射方向。

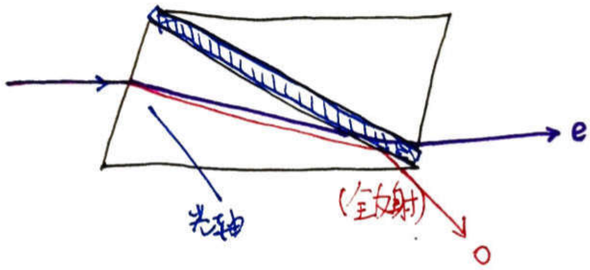
1) 晶体的二向色性.（偏振片）

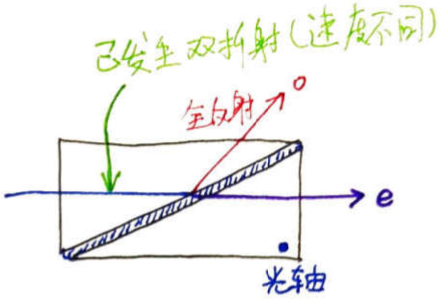
如电气石对o光吸收大，对e光吸收小，可作为偏振片.

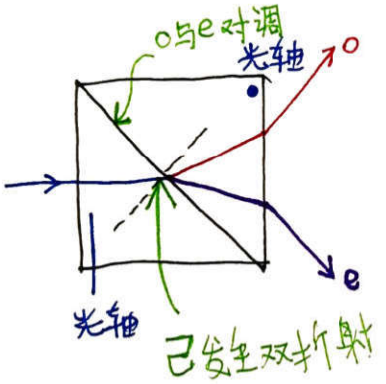
2) 偏振棱镜

① 将e与o分开

② 偏振度高的单束线偏振光

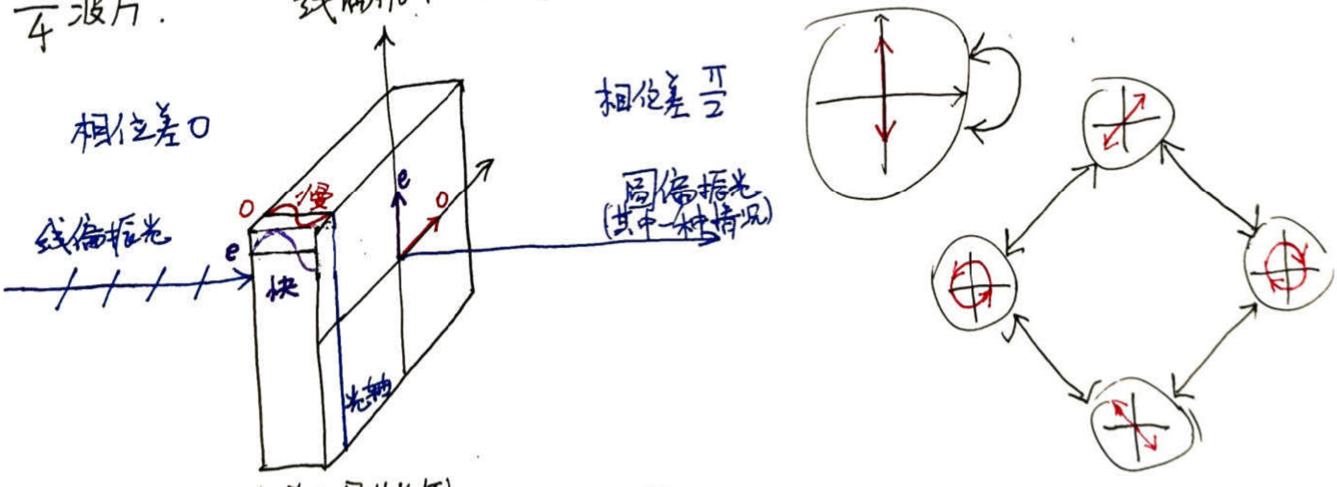






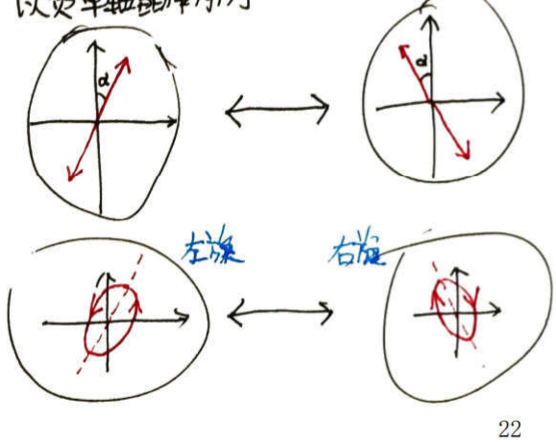
$\frac{1}{4}$波片.

线偏振⟷正椭圆/圆偏振（其中一种情况）

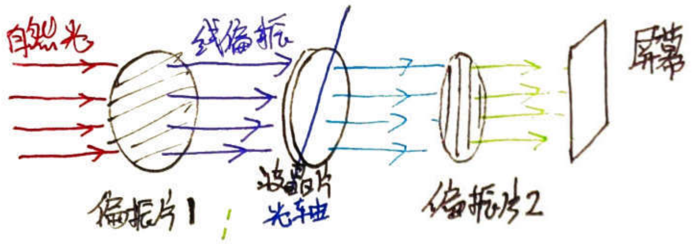


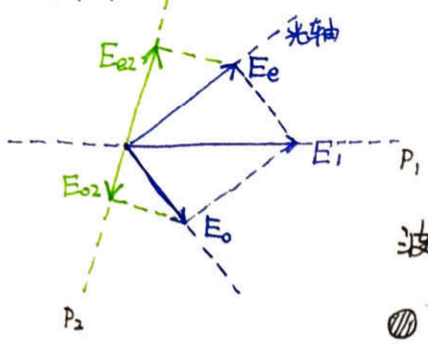
以负单轴晶体为例

$\frac{1}{2}$波片



平行偏振光的干涉





$\tilde{E}_2=\tilde{E}_{e2}+\tilde{E}_{o2}$，$\tilde{E}_{e2}$与$\tilde{E}_{o2}$的相位差 $\delta=\frac{2\pi d}{\lambda}(n_o-n_e)$

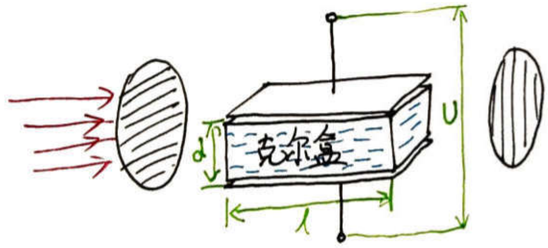
波晶片厚度

波晶片：尖劈型 $\longrightarrow$ 等厚干涉

光测弹性术

电光效应 $<$ 克尔效应
泡克耳斯效应

克尔效应：某些各向同性介质外加电场后具有单轴晶体的特性.



$n_o-n_e=B\cdot\lambda\cdot E^2$

克尔常量　电场强度

注：大多数情况 $n_o-n_e>0$.

B与材质、λ、温度等有关.

$\tilde{E}_{e2}$与$\tilde{E}_{o2}$的相位差：$\delta=\frac{2\pi l}{\lambda}(n_o-n_e)\propto U^2$

旋光性.

线偏振光 $\xrightarrow[\text{(菲涅耳)}]{\text{分解}}$ 左旋圆偏，有 n_L；右旋圆偏，有 n_R

左旋晶体 $n_R>n_L$，$V_L>V_R$，线偏振方向逆时针转；

右旋晶体 $n_L>n_R$，$V_R>V_L$，顺时针转（接收者）

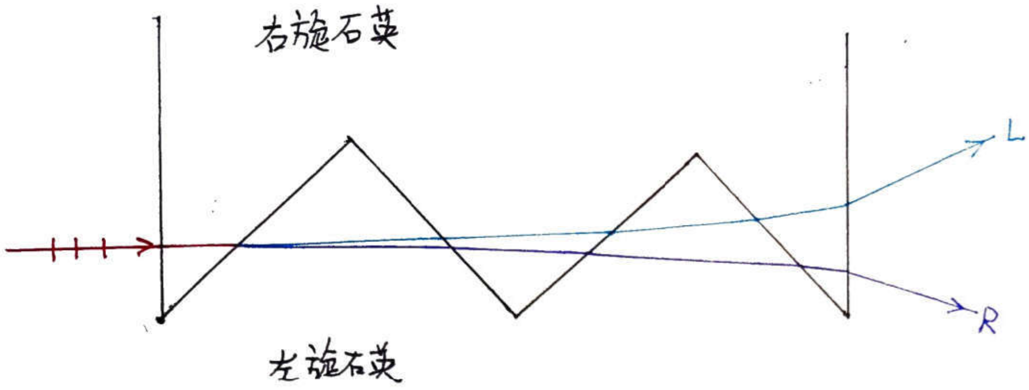
线偏振旋转角 $\varphi=\alpha\cdot l$（顺时针转）

注：射出光仍为线偏振光.

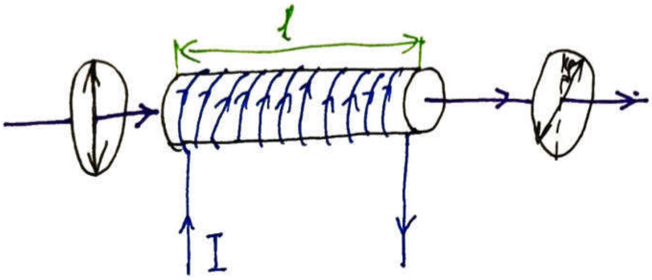
旋光率（与材料、波长等有关）$<$ $\alpha<0$ 左旋（不是左旋光）
$\alpha>0$ 右旋.

$\varphi=[\alpha]\cdot C\cdot l$

溶液的比旋光率　溶液浓度



磁致旋光性（法拉第效应）



韦尔代常量

$\varphi = V \cdot B \cdot l$

磁感应强度

$\begin{cases} V<0 & \text{左旋（逆时针）负旋体} \\ V>0 & \text{右旋（不是右旋光）正旋体} \end{cases}$

仍是线偏振光。

光的吸收（一般光源）

朗伯定律 $I = I_0 e^{-\alpha x}$

α：吸收系数

x：介质厚度

比尔定律 $I = I_0 e^{-\beta C l}$

C：透明介质中溶剂的浓度

βC：吸收系数

注：稀溶液，即介质分子的吸收本领不受周围分子的影响。

复数相对介电常数

若 ε_r 为实数，则 $n = \sqrt{\varepsilon_r}$.

$$E(x,t) = E_0 \exp\{i(kx-\omega t)\} = E_0 \exp\left\{-i\omega\left(t - \frac{x\sqrt{\varepsilon_r}}{c}\right)\right\}$$ 变为负数

复数折射率

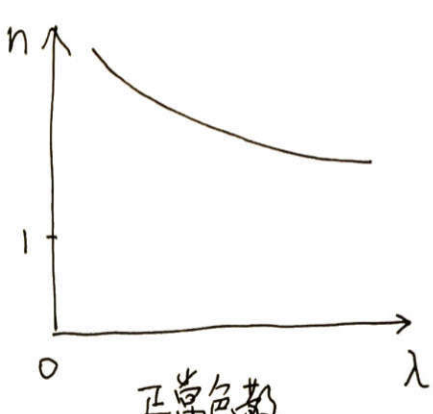
令 $\sqrt{\varepsilon_r} = \tilde{n} = n + in'$

$$E = E_0 \exp\left\{-i\omega\left[t-(n+in')\frac{x}{c}\right]\right\} = E_0 e^{-\frac{\omega}{c}n'x} \exp\left\{-i\omega\left(t - n\frac{x}{c}\right)\right\}$$

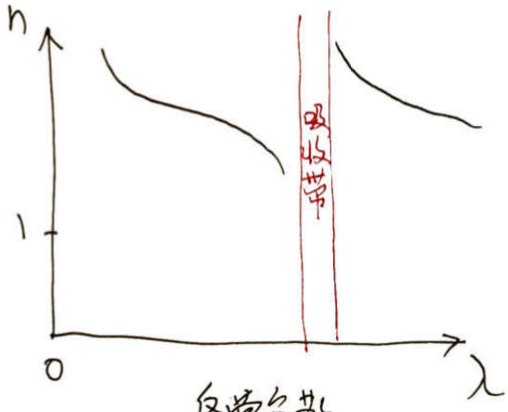
$$\Rightarrow \alpha = \frac{2\omega}{c}n' = \frac{4\pi n'}{\lambda}.$$

柯西方程

$$n^2 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

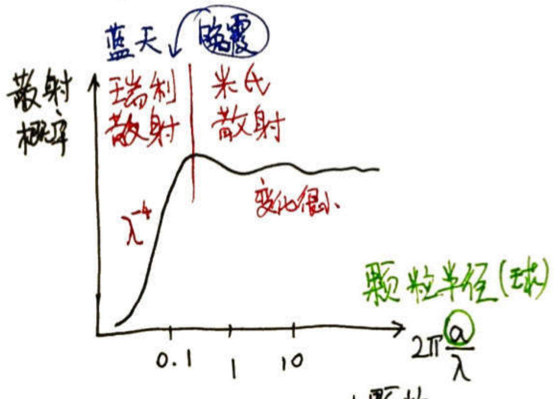


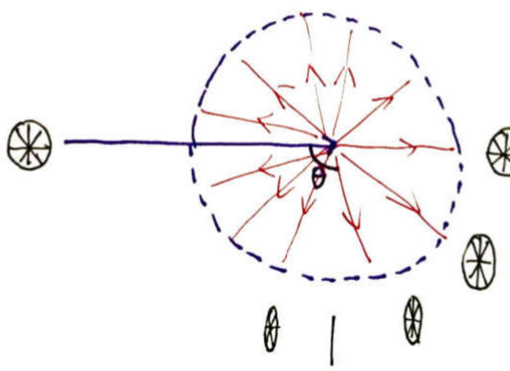
正常色散
（吸收很少的波段）



反常色散
（吸收很强的波段）

光的散射





$$I = I_0(1+\cos^2\theta)$$

→ 大颗粒

→ 短波

海水为什么是蓝色的.

拉曼散射 & 布里渊散射　除 ω_0 散射，还有 $\omega_0 \pm \omega_1$，$\omega_0 \pm \omega_2$，… 等存在

高级　低级

热辐射体的辐通量（辐射功率，单位 W）

光谱辐出度 $r_e(\lambda,T)=\dfrac{d^2\Phi(\lambda,T)}{dS\,d\lambda}$ 单位时间 单位表面积、单位波长 辐出度

$$R_e(T)=\int_0^\infty r_e(\lambda,T)\,d\lambda$$

绝对黑体辐出度 $R_0(T)=\int_0^\infty r_0(\lambda,T)\,d\lambda$

吸收本领 $a(\lambda,T)=\dfrac{\text{吸收辐通量}}{\text{入射辐通量}}$ 绝对黑体 $a=1$

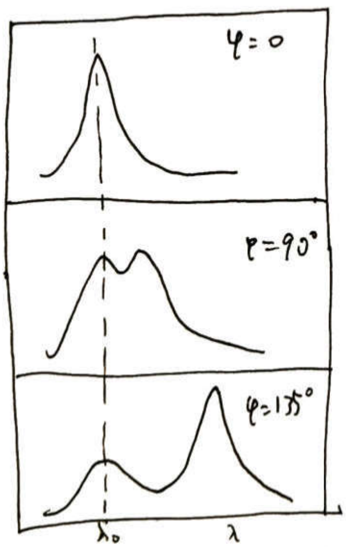
斯特藩-玻耳兹曼定律

$$R_0(T)=\sigma T^4$$

维恩位移定律

$\lambda_m=\dfrac{b}{T}$ 黑体辐射谱曲线极大点 ---- 峰值定律 $r_0(\lambda_m,T)=C'T^5$

康普顿效应（康普顿散射）



康普顿散射的特点.

1. 除λ_0外有长波新波长λ.
2. $\varphi\nearrow$，$\lambda\nearrow$ （$\lambda-\lambda_0=\lambda_c(1-\cos\varphi)$）
   散射角 康普顿波长 $\lambda_c=\dfrac{h}{m_0c}=2.43\times10^{-3}\,nm$
3. $\varphi\nearrow$，新波长谱线强度升高

只有$\lambda_0\sim\lambda_c$时 康普顿效应才显著.（硬X射线）

定性分析. X射线光子与电子碰撞. 弹性碰撞 → 光子把部分能量传给电子

若吸收光子，无法同时满足能量守恒与动量守恒

激光.

自发发射 $-\left(\frac{dN_2}{dt}\right)_{sp} = A_{21} N_2$

自发发射系数

受激吸收 $\left(\frac{dN_2}{dt}\right)_{a} = B_{12} N_1 \cdot w(\nu, T)$

受激吸收系数

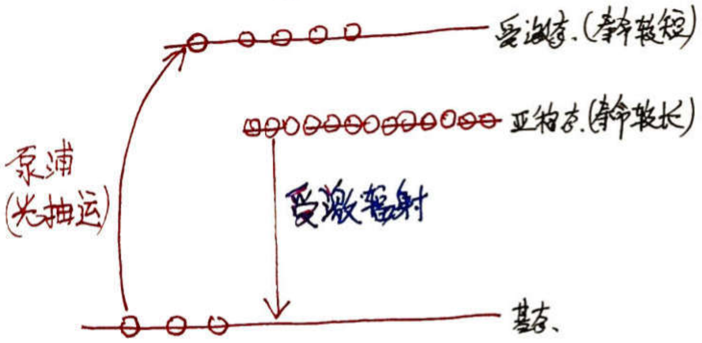
受激发射 $-\left(\frac{dN_2}{dt}\right)_{st} = B_{21} N_2 \cdot w(\nu, T)$

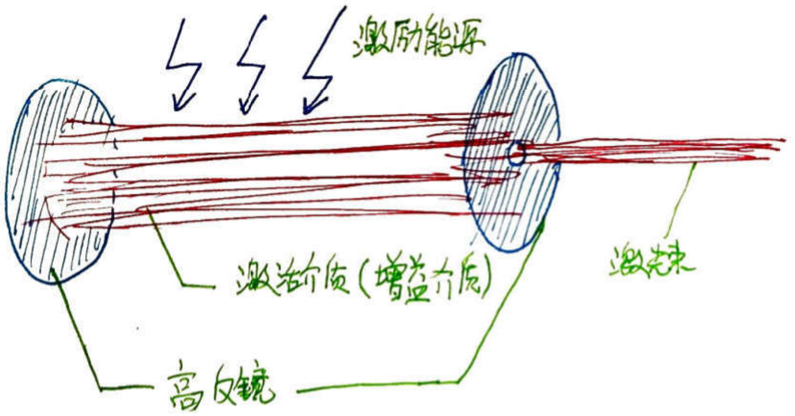
自发发射系数

$h\nu$ 全同光子

光放大的条件：$\boxed{\text{粒子数反转} \quad N_2 > N_1}$

① 需要至少3能级的系统（产生） ② 泵浦源（维持） ③ 光学谐振腔（方向性,单色性 充分放大）





增益系数,与I,ν有关.

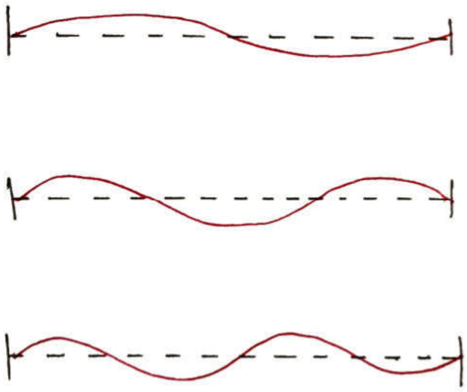
$$I = I_0 e^{Gx}$$

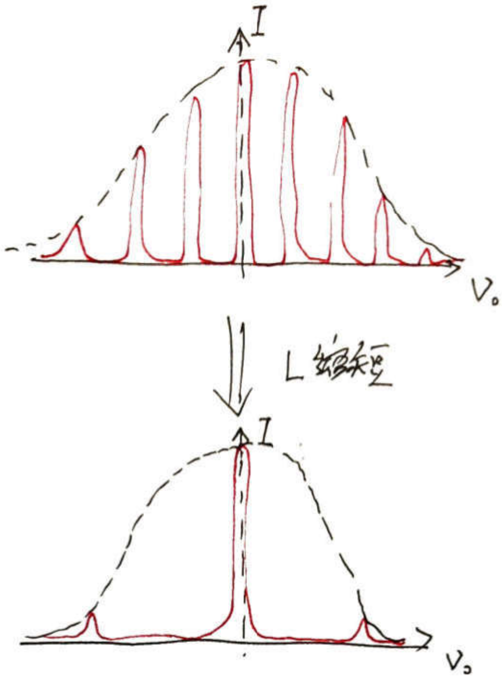
$$G \geq \frac{1}{2L} \ln \frac{1}{R_1 R_2} = G_m \quad \text{阈值条件}$$

反射率(光强)

激光的单色性：多光束干涉

$2nL = k\lambda_0, \ k \in Z^+ \quad \Rightarrow \quad \nu_0 = \frac{kc}{2nL}$ 纵模间隔

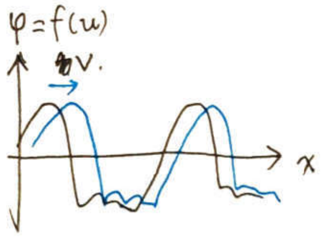




Part 2

同步课程

叶贤基《波动光学》



波形不变，波平移
注：不限于正弦波

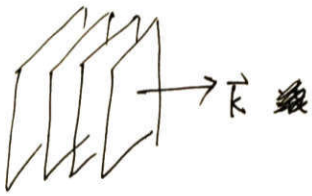
波动方程	三维波动方程
$\frac{\partial^2\varphi}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2\varphi}{\partial t^2}$	$\nabla^2\varphi=\frac{1}{v^2}\frac{\partial^2\varphi}{\partial t^2}$

Phase $\varphi=kx+\omega t+\varphi_0$

Phase Velocity $\left(\frac{\partial x}{\partial t}\right)_\varphi=\pm\frac{\omega}{k}=\pm v$

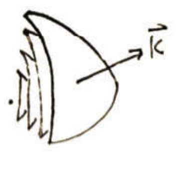
波前 相位相同的点的集合

平面波



$$\psi(\vec{r},t)=Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

球面波



$$\psi(\vec{r},t)=\frac{A}{r}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

E-M Wave Equations

$$\nabla^2\vec{B}-\varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}=0$$

$$\nabla^2\vec{E}-\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=0$$

Poynting vector (Energy flux) = $u c$ 能量的空间密度

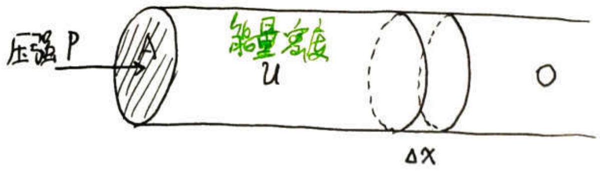
$$\vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}=\varepsilon_0c^2\vec{E}\times\vec{B}$$

$\boxed{\text{定义: } \vec{S}\equiv\vec{E}\times\vec{H}}$

E-M wave 频率太高时，只能测平均功率

即 $I=\frac{1}{2}c\varepsilon_0E^2$

$u\cdot(A\Delta x) = PA\Delta x$　　压强$P \approx$ 能量密度 u.



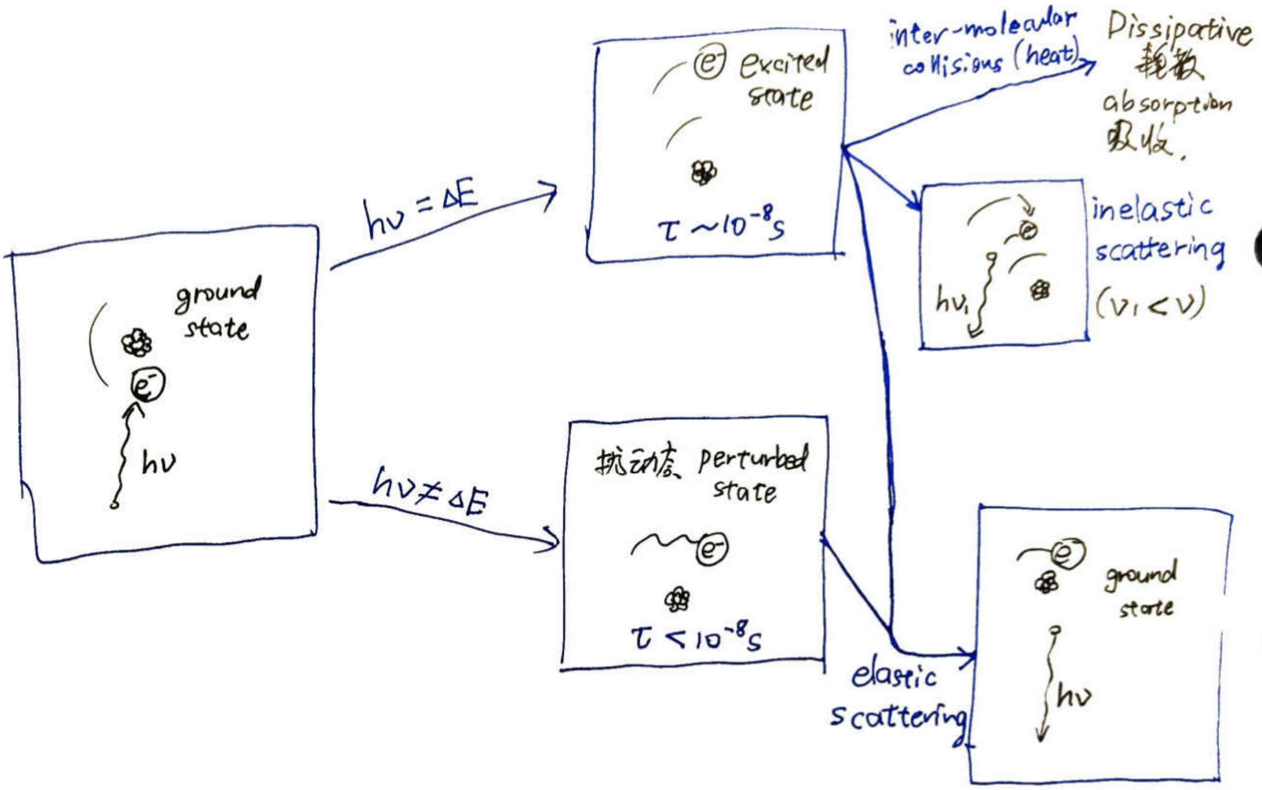
light pressure　　$P = \frac{S}{c}$ (全吸收)

$P = \frac{2S}{c}$ (全反射)

$E^2 = \underbrace{p^2}_{\text{动量}} c^2 + \underbrace{m_0^2}_{\text{静止质量}} c^4 = m^2c^4$

$\Rightarrow$ 光子:

$E = pc = h\nu$.



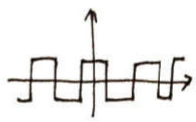
叠加 $\begin{cases} \tilde{E}_1 = E_0 e^{i(k_1 x - \omega_1 t)} \\ \tilde{E}_2 = E_0 e^{i(k_2 x - \omega_2 t)} \end{cases}$

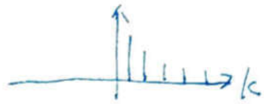
设 $\begin{cases} \bar{k} = \frac{k_1+k_2}{2}, & k_m = \frac{k_1-k_2}{2} \text{ 调制波数} \\ \bar{\omega} = \frac{\omega_1+\omega_2}{2}, & \omega_m = \frac{\omega_1-\omega_2}{2} \text{ 调制频率} \end{cases}$

则 $\tilde{E} = \tilde{E}_1 + \tilde{E}_2 = E_0 \underbrace{e^{i(\bar{k}x - \bar{\omega}t)}}_{拍} \left[e^{ik_m x} e^{-i\omega_m t} + e^{-ik_m x} e^{i\omega_m t} \right]$

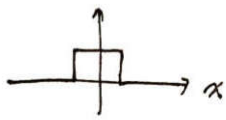
→ 群速度 $v = \bar{\omega}/\bar{k}$.

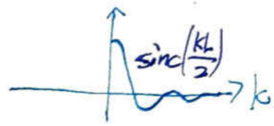
Fourier 级数 — ω 离散

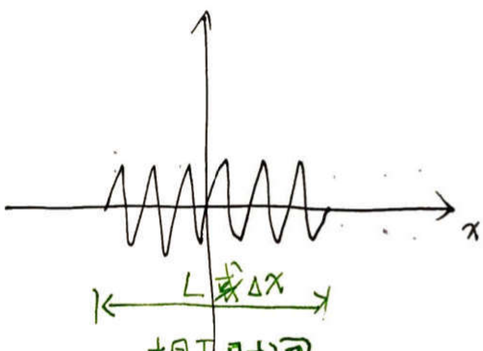


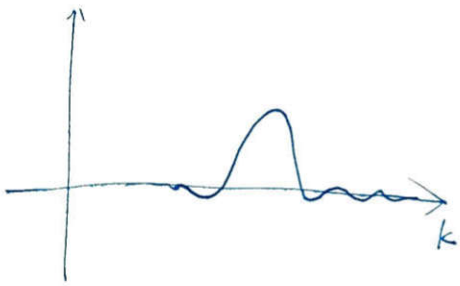


Fourier 变换 — ω 连续









相干时间
相干长度L > 都是时间的相干性

$L_0 \sim \frac{\lambda^2}{\Delta\lambda}$

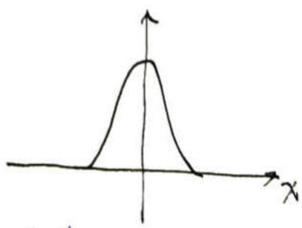
$t_0 = \frac{L_0}{c}$.

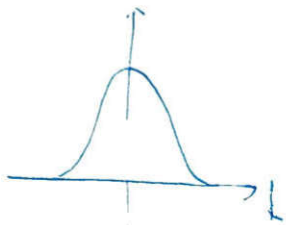
相干：相位差要稳定

注：当 相位差为 2π (我们认为可分辨的上限)

即 $\Delta k \cdot \underbrace{\Delta x}_{相干长度} = 2\pi \ (4\pi)$

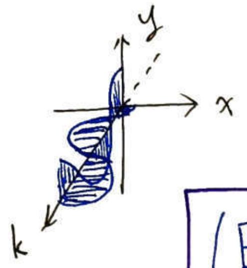
亦可写作 $\Delta\omega \cdot \underbrace{\Delta t}_{相干时间} = 2\pi \ (4\pi)$





高斯函数 $f(x) = \exp\left\{-\frac{x^2}{a^2}\right\}$ $\Longleftrightarrow$ 高斯函数 $f(k) = \frac{a}{2\sqrt{\pi}} \exp\left\{-\frac{a^2 k^2}{4}\right\}$

偏振 Polarization



$$E_x = E_{x_0}\cos\theta \quad , \theta = kx - \omega t$$

$$E_y = E_{y_0}\cos(\theta+\varepsilon)$$

$$\boxed{\left(\frac{E_x}{E_{x_0}}\right)^2 + \left(\frac{E_y}{E_{y_0}}\right)^2 - 2\left(\frac{E_x}{E_{x_0}}\right)\left(\frac{E_y}{E_{y_0}}\right)\cos\varepsilon = \sin^2\varepsilon}$$

偏振态 State of Polarization

P : Plane- or linear-polarized.

☆ R : Right-circular-polarized.

☆ L : Left-circular-polarized.

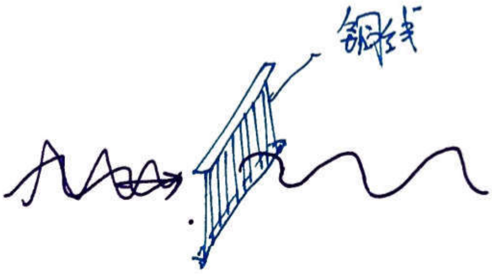
E : Elliptical-polarized.

$$\begin{cases} R = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \\ L = \cos\theta\,\hat{i} - \sin\theta\,\hat{j} \end{cases}$$

$$\begin{cases} P = R \pm L \\ E = aR \pm bL \end{cases}$$

随机偏振光 (√)　非偏振光 (×)

Wire-Grid Polarizer.



穿过的电场垂直于铜线

Dichroism：吸收率与偏振方向有关

(二向色性)

Birefringence 双折射

$$\Delta n = n_e - n_o = \frac{c}{v_{/\!/}} - \frac{c}{v_\perp}$$

$\Delta n < 0$ Negative single-axis crystal

opt. axis = fast axis

$\Delta n > 0$ Positive

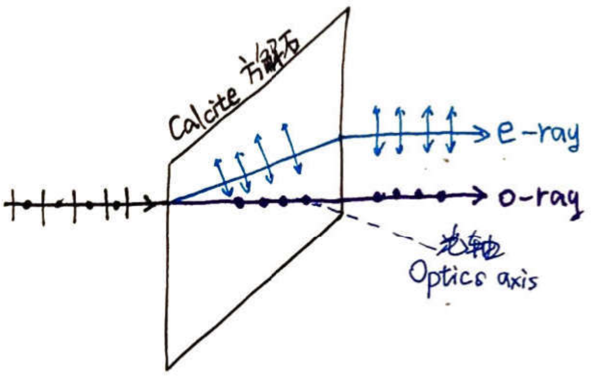
opt. axis = slow axis

$$\boxed{\begin{aligned}&\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\\ &\nabla\times\vec{B} = \mu\vec{J} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\\ &\nabla \sim i\vec{k}\\ &\vec{D} = \varepsilon\vec{E},\ \varepsilon\text{为张量}\end{aligned}} \longrightarrow \vec{D} = \frac{k^2\mu}{\omega^2}\vec{E}_{k\perp} \Rightarrow \vec{D}\perp\vec{k}$$

$\vec{E}$在垂直于$\vec{k}$方向上的分量

偏振方向：

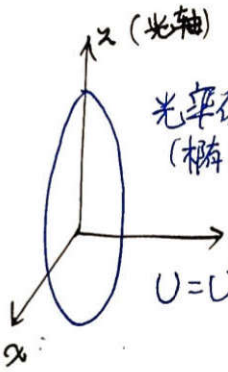
o光 ⊥ 光轴， e光不一定 // 光轴



光轴：$\vec{D} = \varepsilon\vec{E}$ 中，

with ε is diagonalized.

对角化的

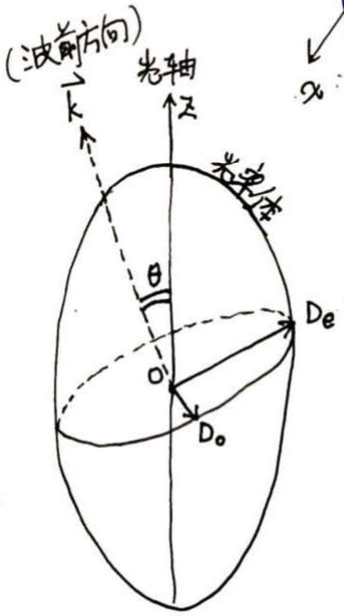


光率体 ellipsoid of wave normal

(椭球)

大小代表功率

$$U = U_e + U_m = \vec{E}\cdot\vec{D} = \frac{D_x^2}{\varepsilon_x} + \frac{D_y^2}{\varepsilon_y} + \frac{D_z^2}{\varepsilon_z}$$



推导步骤

① $\vec{D}\perp\vec{k}$

定出 $\vec{D_o}$ 与 $\vec{D_e}$

$(\vec{D_e}\perp\vec{D_o})$.

② 找出 $\vec{E_e}$ 与 $\vec{S_e} = \vec{E_e}\times\vec{H_e}$

($\vec{H}$方向不变)

$$\begin{cases} D_o = \varepsilon_x E_x\hat{i} + \cancel{\varepsilon_y E_y\hat{j}} + \cancel{\varepsilon_z E_z\hat{k}} \\ D_e = \cancel{\varepsilon_x E_x\hat{i}} + \varepsilon_y E_y\hat{j} + \varepsilon_z E_z\hat{k} \end{cases}$$

$$\Rightarrow \begin{cases} D_o = \varepsilon_o E_x\hat{i} \\ D_e = \varepsilon_o E_y\hat{j} + \varepsilon_e E_z\hat{k} \end{cases}$$

Summary .

$\mu_r=1$, $j_0=0$ 时（平面波）

$$B=\mu_0 H \qquad E=E_0 e^{-i(\omega t-\vec{k}\vec{r})}$$

$$\nabla\times E=-\frac{\partial B}{\partial t} \qquad D=D_0 e^{-i(\omega t-\vec{k}\vec{r})}$$

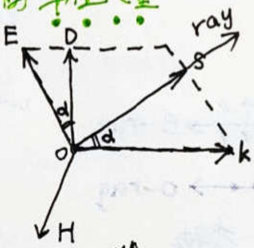
$$\nabla\times H=\frac{\partial D}{\partial t} \qquad H=H_0 e^{-i(\omega t-\vec{k}\vec{r})}$$

Tips: $\nabla \sim i\vec{k}$

其中 $\vec{k}$ 表示波矢方向单位矢量

$$H=\frac{1}{\omega\mu_0}\vec{k}\times E$$

$$D=-\frac{1}{\omega}\vec{k}\times H$$



$$V_{s_e}^2=V_o^2\cos^2\theta+V_e^2\sin^2\theta$$

$$V_{s_o}=V_o$$

ε为(对角化后的)张量

$$D=\varepsilon E$$

$$w_e=\frac{1}{2}D\cdot E$$

电场能量密度

$$w_e=\frac{1}{2\varepsilon_0}\left[\left(\frac{D_x}{\sqrt{\varepsilon_x}}\right)^2+\left(\frac{D_y}{\sqrt{\varepsilon_y}}\right)^2+\left(\frac{D_z}{\sqrt{\varepsilon_z}}\right)^2\right]$$

光率体（介电常量椭球）

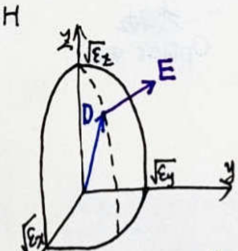
记 $x=\dfrac{D_x}{\sqrt{2\varepsilon_0 w_e}}$

$y=\cdots$, $z=\cdots$

则有

$$f(x,y,z)=\frac{x^2}{\varepsilon_x}+\frac{y^2}{\varepsilon_y}+\frac{z^2}{\varepsilon_z}-1=0$$

且有 $\nabla f \,/\!/\, E$.



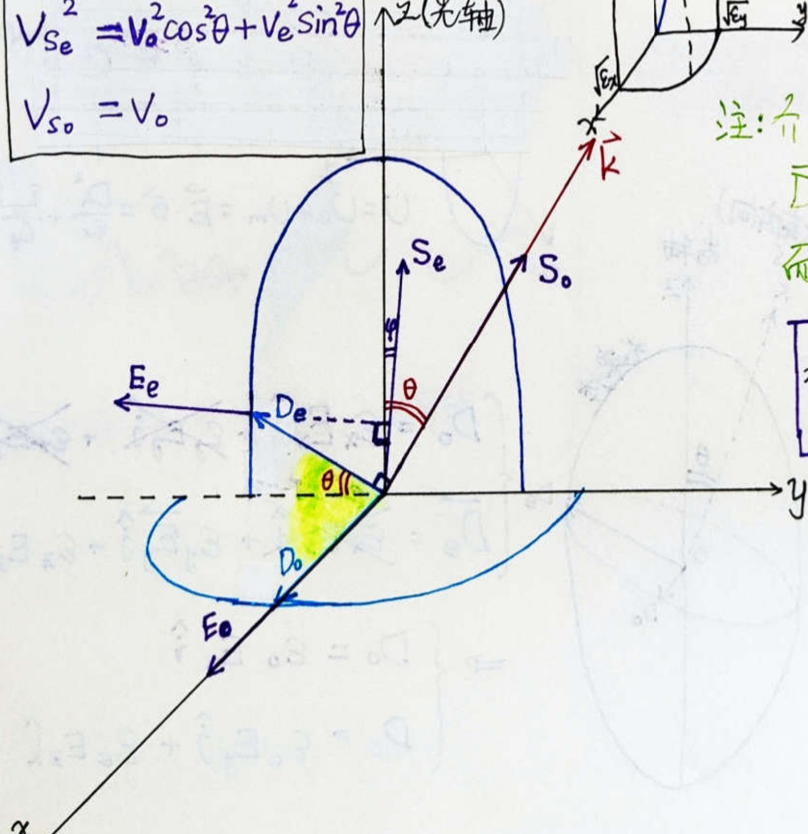
注：介质内电场振动实际为D而非E.

$\vec{D}$ 可分解为 $\vec{D_o}$ 与 $\vec{D_e}$

而 $\vec{k}$ 的方向始终不变，可视为"给定"

$$\boxed{\text{椭球} \Rightarrow \tan\varphi=\frac{\varepsilon_o}{\varepsilon_e}\tan\theta}$$

指o光



Stokes Parameters.（可描述部分偏振光）不常用

$$S_0 = \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle$$

$$S_1 = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle$$

$$S_2 = \langle 2E_{0x}E_{0y}\cos(\varepsilon_x - \varepsilon_y) \rangle$$

$$S_3 = \langle 2E_{0x}E_{0y}\sin(\varepsilon_x - \varepsilon_y) \rangle$$

Unpolarized: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

degree of polarization

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

The Jones Vectors（只能描述全偏振光）常用

$$E = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix} = \cancel{E_{0x} e^{i\varphi_x}} \begin{bmatrix} 1 \\ \frac{E_{0y}}{E_{0x}} e^{i(\varphi_y - \varphi_x)} \end{bmatrix}$$

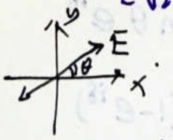
有时忽略

归一化后的 Jones Vectors 为：

更本质

Hori. P-state	Vert. P-state	P-state at +45°	P-state at −45°	R-state	L-state
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$

例：用 R 与 L 表示的线性叠加



$$E = \begin{bmatrix} E_0 \cos\theta \; e^{i\varphi} \\ E_0 \sin\theta \; e^{i\varphi} \end{bmatrix} \sim \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ i \end{bmatrix} + \beta \begin{bmatrix} 1 \\ -i \end{bmatrix} \Rightarrow \begin{cases} \alpha = \dfrac{\cos\theta - i\sin\theta}{2} = \dfrac{1}{2} e^{-i\theta} \\ \beta = \dfrac{\cos\theta + i\sin\theta}{2} = \dfrac{1}{2} e^{i\theta} \end{cases}$$

注：矩阵转置时复数要取共轭.

器件的 Jones Matrices

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = J \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

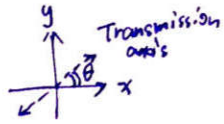
出射　　　　入射

Horizontal 线偏振片: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Vertical: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

+45° 线偏振片: $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

-45°: $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$



$$\begin{bmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \frac{1}{2}\sin 2\theta & \sin^2\theta \end{bmatrix}$$

$\frac{1}{4}$ 波片

小n，小光程，小相位改变
(OPL)

fast axis vertical: $e^{\frac{i\pi}{4}}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ 　可去

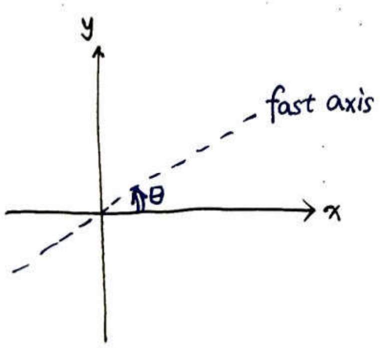
fast axis horizontal: $e^{\frac{i\pi}{4}}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ 　可去

Homogeneous circular polarizer right: $\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$

left: $\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$

相差为δ时 (fast axis horizontal)

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \text{ with } \delta = \frac{2\pi d}{\lambda}\Delta n.$$



$$J = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta\, e^{i\delta} & \sin\theta\cos\theta(1-e^{i\delta}) \\ \sin\theta\cos\theta(1-e^{i\delta}) & \sin^2\theta + \cos^2\theta\, e^{i\delta} \end{bmatrix}$$

Interference 干涉

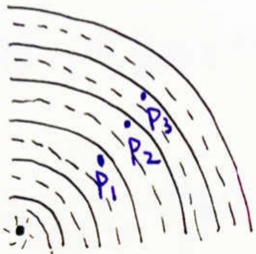
— 空间相干 条纹　　波前不畸变

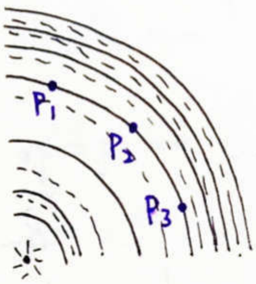
— 时间相干 拍频　　频率稳定

- 相干时间
- 相干长度

干涉的条件: ① 偏振: 不正交

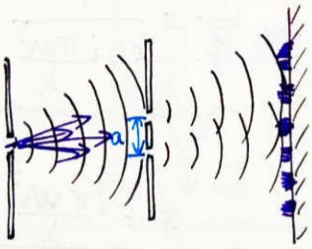
② 相干: 时间相干 & 空间相干.





时间相干:

空间相干: P_1、P_2、P_3 的相位高度关联



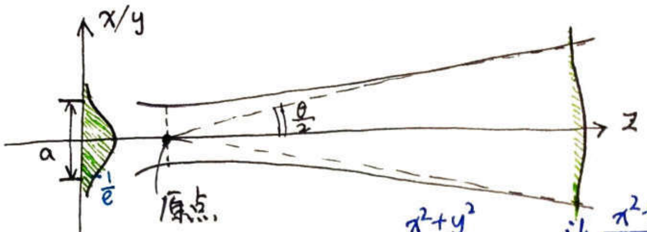
$\underbrace{\Delta x}_{a} \cdot \Delta p_x \sim h$,　　出射偏转角 $\theta \sim \frac{\Delta p_x}{p}$, $p = \frac{h}{\lambda}$

$\Rightarrow \theta \sim \frac{\lambda}{a}$

粒子性

若时间相干性不佳(频率不稳定)
则条纹不断变化。

Gaussian beam 高斯光束



$$E \propto \underbrace{E_0}_{\text{原点场强}} \cdot e^{ikz} \cdot \underbrace{e^{-\frac{x^2+y^2}{W}}}_{\text{Gaussian 函数}} \cdot e^{ik\frac{x^2+y^2}{2R}} \cdot \underbrace{e^{-i\arctan\frac{z}{z_R}}}_{z\to\infty\text{时可忽略}} \cdot \underbrace{\frac{1}{R}}_{\text{随距离衰减}}$$

$$e^{ik\left(z+\frac{x^2+y^2}{2R}\right)} \sim \frac{e^{ikr}}{R}$$

Using $q \in \mathbb{C}$ to replace z, and $q = \underbrace{q_0}_{\text{纯虚}} + \underbrace{z}_{\text{实}}$.

Assume $\frac{1}{q} = \frac{1}{R} + i\frac{\lambda}{\pi W^2}$ ① $\Rightarrow \frac{1}{q_0+z} = \frac{1}{R} + i\frac{\lambda}{\pi W^2}$.

when $z=0$, $R\to\infty$, $W=W_0$ $\Rightarrow q_0 = \frac{\pi W_0^2}{i\lambda}$ $\Rightarrow \frac{1}{q} = \frac{1}{z - i\frac{\pi W_0^2}{\lambda}}$ ②

$$= \frac{z + i\frac{\pi W_0^2}{\lambda}}{z^2 + \frac{\pi^2 W_0^4}{\lambda^2}}$$

①② 比较实部 $\Rightarrow R = z\left(1+\frac{z_R^2}{z^2}\right) \longrightarrow z$ 波前曲率半径

with Rayleigh range $z_R := \frac{\pi W_0^2}{\lambda}$

①② 比较虚部 $\Rightarrow W = W_0\sqrt{1+\frac{z^2}{z_R^2}} \longrightarrow \frac{W_0 z}{z_R}$ 光斑半径

$$\Rightarrow \theta = \frac{W}{z} \sim \frac{W_0}{z_R} = \frac{1}{\pi}\frac{\lambda}{W_0} \sim 1.22\frac{\lambda}{D}.$$

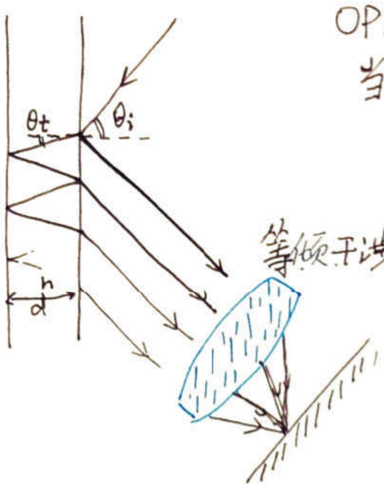
$$E \propto u \sim \frac{e^{i\frac{k(x^2+y^2)}{2q}+ikz}}{q} = \left(\frac{1}{R} + i\frac{\lambda}{\pi W^2}\right) e^{i\frac{k(x^2+y^2)}{2}\left(\frac{1}{R}+i\frac{\lambda}{\pi W^2}\right)} \xrightarrow{k=\frac{2\pi}{\lambda}} \cdot e^{ikz}$$

$$= \left(\sqrt{\cancel{\frac{1}{R^2}} + \frac{\lambda^2}{\pi^2 W^4}}\ \cancel{e^{i\arctan\frac{\lambda R}{\pi W^2}}}\right) \cdot e^{i\frac{k(x^2+y^2)}{2R}} \cdot e^{-\frac{x^2+y^2}{W^2}} \cdot e^{ikz}$$

亦可不去掉

$$\xrightarrow{\text{远场}} \frac{\lambda}{\pi W^2} e^{-\frac{x^2+y^2}{W^2}} e^{i\phi(x,y,z)},$$ with $\phi(x,y,z) = \frac{k(x^2+y^2)}{2R} + kz$

Amplitude-Splitting Interferometers 分振幅干涉

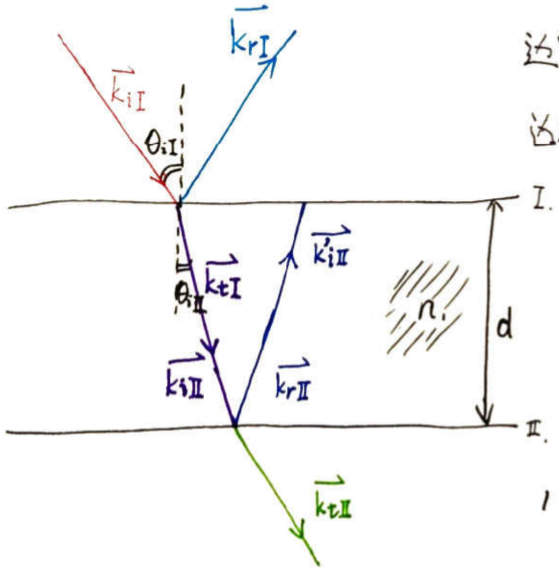


$OPD = 2nd \cdot \cos\theta_t$

当 $\theta_i < \theta_B$, phase shift $= 2nd \cdot \cos\theta_t \pm \pi$.

等倾干涉：d固定，入射角一致的光干涉.

F-P 腔 (见基础光学笔记)



边界条件：$\vec{E}$ 与 $\vec{H} = \frac{\vec{B}}{\mu}$ 的切向分量连续

边界I：$E_I = E_{iI} + E_{rI} = E_{tI} + E'_{rII}$

边界II：$E_{II} = E_{iII} + E_{rII} = E_{tII}$

传播条件：

光程 $h = \frac{n_1 d}{\cos\theta_{iII}}$

$E_{iII} = E_{tI} e^{ik_0 h}$ $\quad E'_{iII} = E_{rII} e^{ik_0 h}$

where $k_0 = \frac{2\pi}{\lambda_0}$

1° E⊥入射面时,

$$H_I = \sqrt{\frac{\varepsilon_0}{\mu_0}}(E_{iI} + E_{rI}) n_0 \cos\theta_{iI}$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}}(E_{tI} - E'_{rII}) n_1 \cos\theta_{iII}.$$

$$H_{II} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdots$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdots$$

...

$$\Rightarrow \begin{bmatrix} E_I \\ H_I \end{bmatrix} = \boxed{\begin{bmatrix} \cos k_0 h & -i\frac{\sin k_0 h}{\gamma_1} \\ -i\gamma_1 \sin k_0 h & \cos k_0 h \end{bmatrix}} \begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix}$$

$$\gamma_1 = \sqrt{\frac{\varepsilon_0}{\mu_0}} n_1 \cos\theta_{iII}.$$

$$\begin{bmatrix} E_{\mathrm{I}} \\ H_{\mathrm{I}} \end{bmatrix} = M_1 \begin{bmatrix} E_{\mathrm{II}} \\ H_{\mathrm{II}} \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} E_p \\ H_p \end{bmatrix} = M_p \begin{bmatrix} E_{p+1} \\ H_{p+1} \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} E_{\mathrm{I}} \\ H_{\mathrm{I}} \end{bmatrix} = M_1 \cdots M_p \begin{bmatrix} E_{p+1} \\ H_{p+1} \end{bmatrix},$$

with $\gamma_1 = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_1\, \cancel{\cos(\ )}$ (垂直入射时).

$$\vdots$$

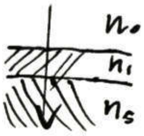
$$\gamma_p = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_p\, \cancel{\cos\theta}$$

$$r = \frac{E_{r\mathrm{I}}}{E_{i\mathrm{I}}}, \quad t = \frac{E_{tp}}{E_{i\mathrm{I}}}, \quad \begin{bmatrix} E_{i\mathrm{I}} \pm E_{r\mathrm{I}} \\ (E_{i\mathrm{I}} \mp E_{r\mathrm{I}})\gamma_0 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_{t\mathrm{II}} \\ E_{t\mathrm{II}}\gamma_s \end{bmatrix}$$

with $\gamma_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_0$, $\gamma_s = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_s$.

$\Rightarrow$ 反射系数 $r = \cdots$, $t = \cdots$

反射率 $R = |r^2| = r^* r$.



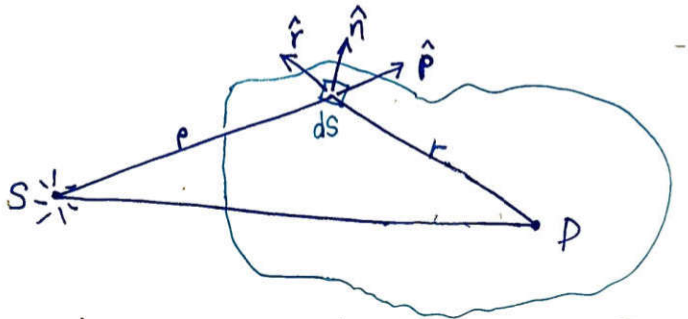
当 $k_0 n_1 d = \frac{\pi}{2}$ 即 $d = \frac{\lambda_1}{4}$，且 $n_1^2 = n_0 n_s$ $\Longrightarrow R_1 = 0$. 增透膜

双层 $\frac{\lambda}{4}$. 且 $\left(\frac{n_2}{n_1}\right)^2 = \frac{n_s}{n_0} \Rightarrow R_2 = 0$ 增透膜.

Kirchhoff's Integral Theorem

$$4\pi E_p = \oint\!\!\oint_S \frac{e^{ikr}}{r} \nabla E \cdot d\vec{S} - \oint\!\!\oint_S E \nabla\left(\frac{e^{ikr}}{r}\right) \cdot d\vec{S}$$

证: $\left\{ \begin{array}{l} \nabla^2 E = \dfrac{1}{c^2}\dfrac{\partial^2 E}{\partial t^2} \\ \text{格林定理.} \end{array} \right.$



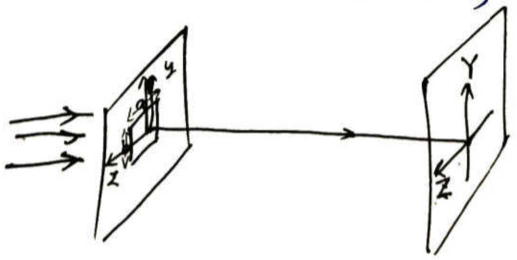
当 $\rho \gg \lambda$, $r \gg \lambda$ 时, 有菲涅耳-基尔霍夫衍射公式:

$$E_p = -\frac{\varepsilon_0 i}{\lambda} \oint\!\!\oint_S \frac{e^{ik(\rho+r)}}{\rho r} K \, ds.$$

其中 倾斜因子 $K = \dfrac{\cos\langle \hat{n}, \hat{r} \rangle - \cos\langle \hat{n}, \hat{\rho} \rangle}{2}$.

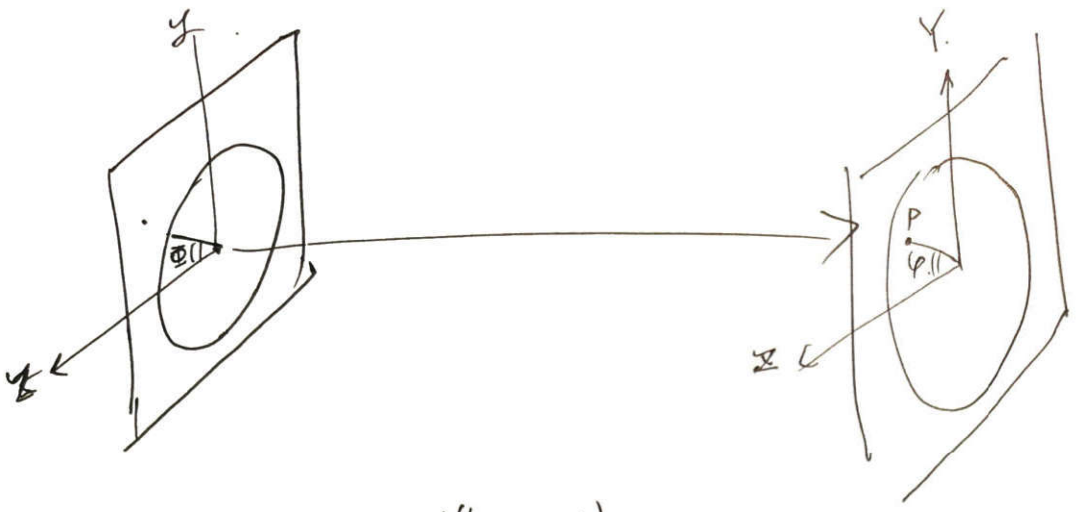
当 $\hat{n}$ 与 $\hat{\rho}$ 反向, $K = \dfrac{\cos\theta + 1}{2}$, $\theta = \langle \hat{\rho}, \hat{r} \rangle$.

远场衍射中, $K \approx 1$. $r \sim R\left(1 - \dfrac{yY + zZ}{R^2}\right)$



$$I(Y,Z) = I(0)\left(\frac{\sin\alpha'}{\alpha'}\right)^2 \left(\frac{\sin\beta'}{\beta'}\right)^2$$

$$\alpha' = \frac{kaZ}{2R} \qquad \beta' = \frac{kbY}{2R}$$



$$dE = \frac{\mathcal{E}_0}{r} e^{i(kr-\omega t)} dS \quad , \quad \underline{r \approx R\left[1-(yY+zZ)/R^2\right]}$$

（远场近似）.

$$yY + zZ = ik(\rho q \sin\phi \sin\Phi + \rho q \cos\phi \cos\Phi)$$

$$= ik\rho q \cos(\phi - \underbrace{\Phi}_{取0})$$

$$= ik\rho q \cos\phi \ .$$

$$\Rightarrow E_P = \iint_{\text{aperture}} dE = \iint_{\text{aperture}} \left(\frac{\mathcal{E}_0}{r}\right) e^{i(kr-\omega t)} dS$$

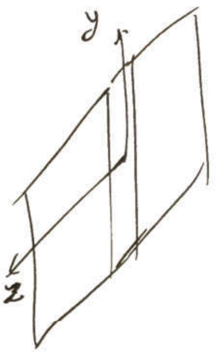
$$\approx \frac{\mathcal{E}_0}{R} e^{i(kR-\omega t)} \iint_{\text{aperture}} e^{ik(yY+zZ)/R} dS \ .$$

$\begin{cases} x = \rho\cos\phi, \\ y = \rho\sin\phi, \end{cases}$ $\begin{cases} Z = q\cos\Phi, \\ Y = q\sin\Phi, \end{cases}$

$$= \frac{\mathcal{E}_0}{R} e^{i(kR-\omega t)} \underbrace{\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} e^{i\left(\frac{k\rho q}{R}\right)\cos\phi}}_{\text{Bessel.}} \rho\, d\rho\, d\phi$$

$$\sim 2\pi \left(\frac{R}{kq}\right)^2 \left(\frac{kqa}{R}\right) J_1\left(\frac{kqa}{R}\right) .$$

单缝情形.



$$r^2=R^2+z^2-2Rz\cos\left(\frac{\pi}{2}-\theta\right).$$

$$\Rightarrow r\sim R-z\sin\theta.$$