原子物理笔记

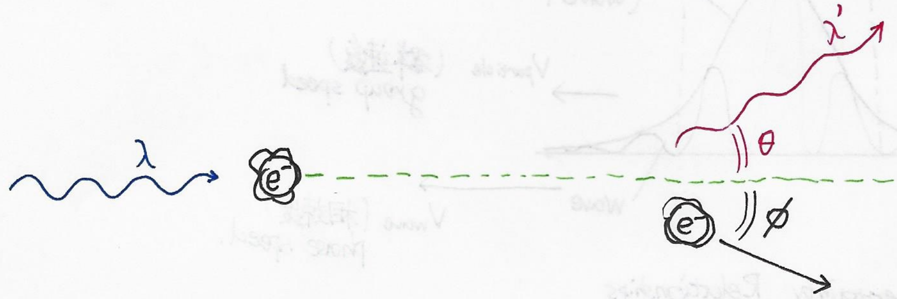
21305337 黄日 2023春季学期

授课老师：罗乐

光子动量 $p=\frac{h}{\lambda}=\frac{h\nu}{c}=\frac{\hbar\omega}{c}=\hbar k$

$\vec{p}=\hbar\vec{k}$

康普顿效应



$$\lambda'=\lambda+\lambda_c(1-\cos\theta)$$

$$\lambda_c=\frac{h}{m_e c}$$

X ray

$\theta=0 \longrightarrow \lambda'=\lambda$

$\theta=\frac{\pi}{2} \longrightarrow \lambda'=\lambda+\lambda_c$

$\theta=\pi \longrightarrow \lambda'=\lambda+2\lambda_c$

里德伯公式：(经验公式).

$$\frac{E}{hc} = \frac{1}{\lambda} = \underbrace{\nu}_{\text{波数}} = \underbrace{R_H}_{\text{里德伯常数}}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

玻尔半径

$a_0 = 0.0529\ nm$

氢原子基态能量

$E_0 = -13.60\ eV$

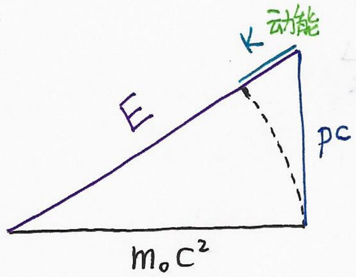
德布罗意：实物粒子具有波粒二象性.

$$\underbrace{\lambda_d}_{\text{德布罗意波长}} = \frac{h}{\underbrace{p}_{\text{动量}}}$$

$K = \frac{1}{2} m v_{particle}^2$

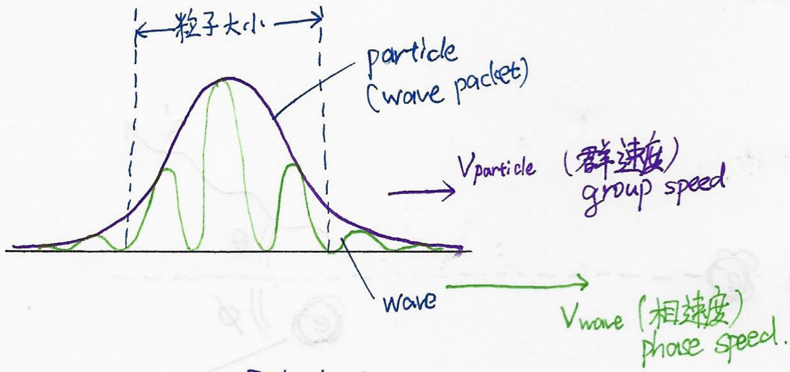
$p = m v_{particle}$

$f\lambda_d = v_{wave}$



$E = \gamma m_0 c^2$.

$$\boxed{E^2 = m_0^2c^4 + p^2c^2}$$



Heisenberg Uncertainty Relationships.

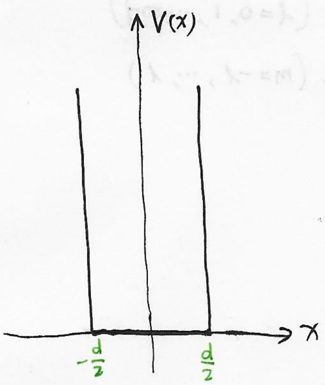
$\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$.

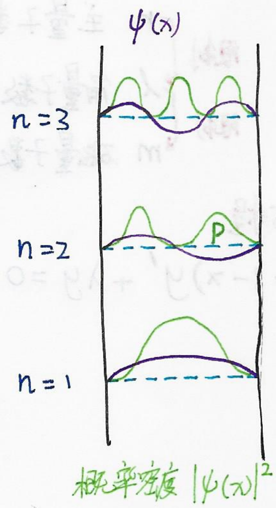
$\Delta t \cdot \Delta E$

定态薛定谔方程

$$\underbrace{\left(-\frac{\hbar^2}{2m}\nabla^2 + \underbrace{V(\vec{x})}_{\text{势能}}\right)}_{\text{哈密顿算符 } \hat{H}} \underbrace{\psi(x)}_{\text{波函数}} = \underbrace{E}_{\text{能量(哈密顿量)}}\psi(x) \longrightarrow \psi'' = -\frac{2m}{\hbar^2}(E-U)\psi$$

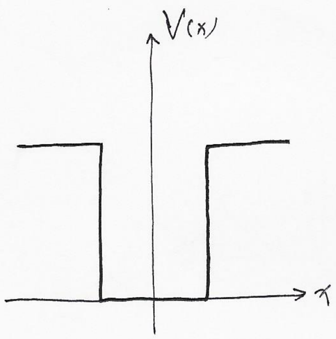
例：一维无穷深势阱

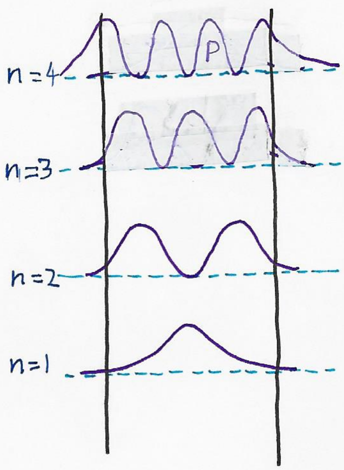




λ_d	$p=\hbar k=\frac{h}{\lambda_d}$	$E=\frac{p^2}{2m}$
$\frac{2d}{n}$	$\frac{nh}{2d}$	$\frac{n^2\pi^2\hbar^2}{2md}$
d	$\frac{h}{d}$	
$2d$	$\frac{h}{2d}$	

例：一维有限深势阱





氢原子的薛定谔方程解.

z轴并不在物理意义上有特殊性，而与测量有关（量子性）.

$$\psi(r,\theta,\phi) = R(r)\ Y(\theta,\phi)$$
$$= R(r)\ \Theta(\theta)\ \Phi(\phi),$$

（未归一化） $R_{n,l}(r) = L_{n+l}^{2l+1}\left(\frac{2r}{na_1}\right)$ 　a_1为玻尔(第一)半径.

$\Theta_{l,m}(\theta) = P_l^m(\cos\theta)$

$\Phi_m(\phi) = e^{im\phi}$

n 主量子数 $(n=1,2,\cdots)$

限制

l 角量子数 $(l=0,1,\cdots,n-1)$

限制

m 磁量子数 $(m=-l,\cdots,l)$

关联拉盖尔 (associated Laguerre) 方程

$$xy'' + (\mu+1-x)y' + \lambda y = 0$$

的解为关联拉盖尔多项式

$$L_\lambda^\mu(x)$$

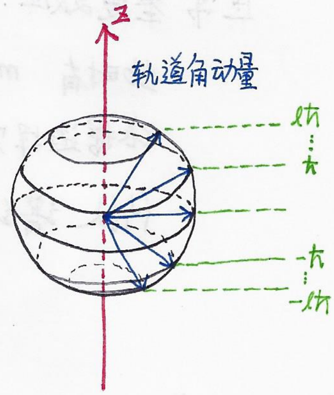
orbital AM

轨道角动量 $L=\sqrt{l(l+1)}\,\hbar \quad (l=0,1,\cdots,n-1)$

z分量 $L_z = m_l\hbar, \ (m_l=0,\pm1,\cdots,\pm l)$

轨道磁矩 $\mu_l = -\sqrt{l(l+1)}\,\mu_B$. 玻尔磁子 $\mu_B = \frac{e\hbar}{2m_e}$.

z分量 $\boxed{\mu_{l,z} = -m_l\,\mu_B}$ ☆



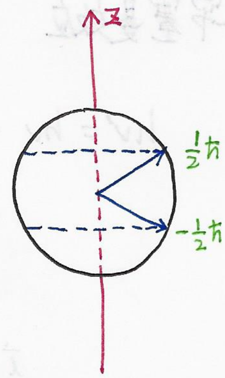
spin AM

自旋角动量 $S=\sqrt{s(s+1)}\,\hbar$, $s=\frac{1}{2}$

z分量 $S_z = m_s\hbar, \ (m_s=\pm\frac{1}{2})$

自旋磁矩 $\mu_{s,} = -2\sqrt{s(s+1)}\,\mu_B$

z分量 $\boxed{\mu_{s,z} \quad -2m_s\,\mu_B}$ ☆



$\updownarrow^{B}_{\mu_s}$ 与 $\Uparrow^{B}_{\mu_s}$ 的 $\Delta E = m_e c^2 \cdot \frac{\alpha^4}{n^5} = 0.511\,\mathrm{MeV}\cdot\frac{\alpha^4}{n^5}$,

其中 fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$

total AM (LS耦合)

总角动量 $\vec{J} = \vec{L} + \vec{S}$, $J = \sqrt{j(j+1)}\,\hbar$, $j = |l-s|,\cdots,|l+s|$

$J_z = m_j\hbar$, $m_j = m_l + m_s$

定义 朗德因子 g, $\begin{cases} g_l = 1 \\ g_s = 2 = 2(1+\frac{\alpha}{2\pi}+\cdots) \end{cases}$, 那么 $\begin{cases} \mu_l = -\sqrt{l(l+1)}\,g_l\,\mu_B \\ \mu_{l,z} = -m_l\,g_l\,\mu_B \\ \mu_s = -\sqrt{s(s+1)}\,g_s\,\mu_B \\ \mu_{s,z} = -m_s\,g_s\,\mu_B \end{cases}$ $\begin{cases} \mu_j = -\sqrt{j(j+1)}\,g_j\,\mu_B \\ \mu_{j,z} = -m_j\,g_j\,\mu_B \end{cases}$

那么 g 的定义式为 $g_A = \frac{\mu_{A,z}/\mu_B}{A_z/\hbar}$ $(A = L, S, J, \cdots)$

$$\begin{cases} \vec{\mu_l} + \vec{\mu_s} = \vec{\mu_j} \\ \vec{L} + \vec{S} = \vec{J} \end{cases} \Rightarrow g_J = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

正常塞曼效应.（不考虑电子自旋，取 $s=0$）

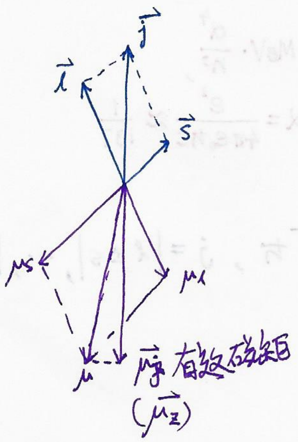
此时有 $m_j = m_l$, $j=l$, $g_j = 1$.

依据选择定则, $\Delta m = 0, \pm 1$, 又因为 $E_0 \longrightarrow E_0 + \underbrace{g_j}_{1} m_j \mu_B B$

所以谱线在磁场中一分为三: $h\nu' \longrightarrow h\nu + \begin{cases} 0 \\ \pm \mu_B B \end{cases}$.

反常塞曼效应（考虑电子自旋）

$$h\nu' = h\nu + (m_2 g_2 - m_1 g_1)\mu_B B .$$



$\vec{j} = \vec{l} + \vec{s}$ 决定 $\vec{\mu}$ 的方向.

考虑S：精细。
考虑I：超精细

原子态：由所有电子轨道合成的原子态

量子数有11个：

n	l	m_l	s	m_s	j	m_j	I	m_I	F	m_F
e^- orbital			e^- spin		e^- total		nuclei-spin		total	

$\vec{J}=\vec{L}+\vec{S}$, $\vec{F}=\vec{I}+\vec{J}$.

$\boxed{n}^{\boxed{2S+1}}\boxed{l}_{\boxed{j}}$

例如 Li-6基态：$2^2S_{1/2}$

$l=0 \rightarrow s$
$1 \rightarrow p$
$2 \rightarrow d$
$3 \rightarrow f$
g
h
i
k

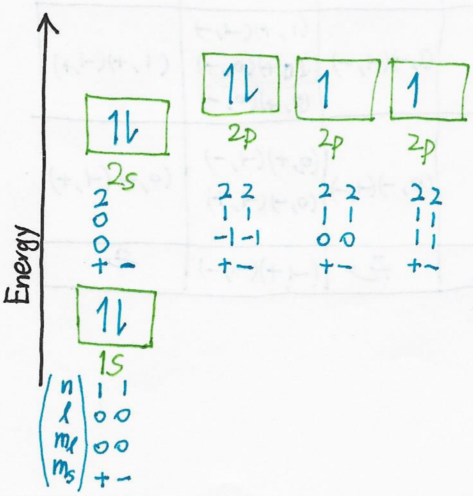
电子组态：由每个电子轨道决定的电子组态

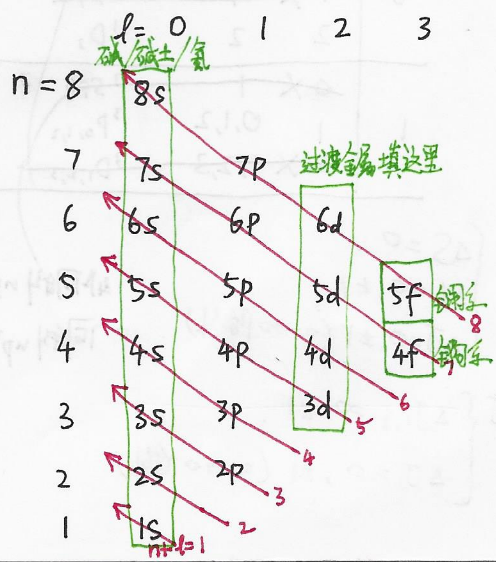
量子数有5个：

n	l	m_l	s	m_s
能级	orb. AM	z-orb AM	spin AM	z-spin AM

$\boxed{n}\boxed{l}^{\boxed{\#e}}$

例如 Li-6基态：$1s^2 2s^1$
B：$1s^2 2s^2 2p^1$

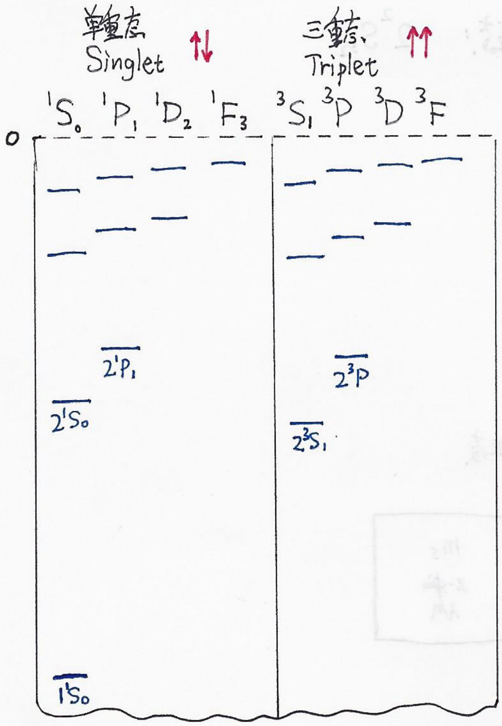


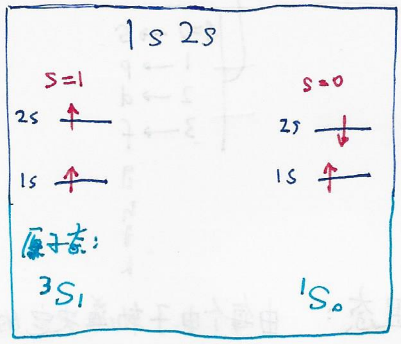


双电子原子

注：2个电子在同轨道时，spin 必反
不同轨道时，spin 可同可反。

spin 同（$S=1$）、spin 反（$S=0$），各自组成一套能级（没有发现之间的跃迁）





同科电子：n, l 都相同的电子

非同科电子：n, l 有不同的电子

同科电子偶数定则：2个电子的LS耦合，必须满足 $L+S$ =偶数。

同科电子的 LS耦合模型 —— 以 $2p^2$ 为例

$(\vec{J}=\vec{L}+\vec{S})$

S (s_1+s_2)	L (l_1+l_2)	J	原子态符号
0	0	0	$^1S_0$
	~~1~~ ×	~~1~~	~~$^1P_1$~~
	2	2	$^1D_2$
1	~~0~~ ×	~~1~~	~~$^3S_1$~~
	1	0,1,2	$^3P_{0,1,2}$
	~~2~~ ×	~~1,2,3~~	~~$^3D_{1,2,3}$~~

非同科 npnp 可用.
同科 np^2 不可用

而 $M_L=0,\pm1,\pm2$ $\quad M_S=0,\pm1$.

M_L ($m_{l1}+m_{l2}$) \ M_S ($m_{s1}+m_{s2}$)	-1	0	$+1$
+2	无	(1,+)(1,−)	无
+1	(1,−)(0,−)	(1,+)(0,−) (1,−)(0,+)	(1,+)(0,+)
0	(1,−)(−1,−)	(1,+)(−1,−) (1,−)(−1,+) (0,+)(0,−)	(1,+)(−1,+)
−1	(0,−)(−1,−)	(0,+)(−1,−) (0,−)(−1,+)	(0,+)(−1,+)
−2	无	(−1,+)(−1,−)	无

m_l m_s

选择定则

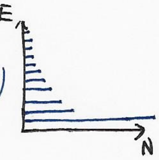
LS $\begin{cases} \Delta S=0 \\ \Delta L=0,\pm1 \\ \Delta J=0,\pm1 \ (0\to0\text{除外}) \end{cases}$

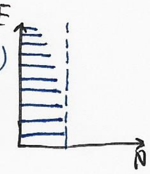
JJ $\begin{cases} \Delta J_{1,2}=0,\pm1 \\ \Delta J=0,\pm1 \ (0\to0\text{除外}) \end{cases}$

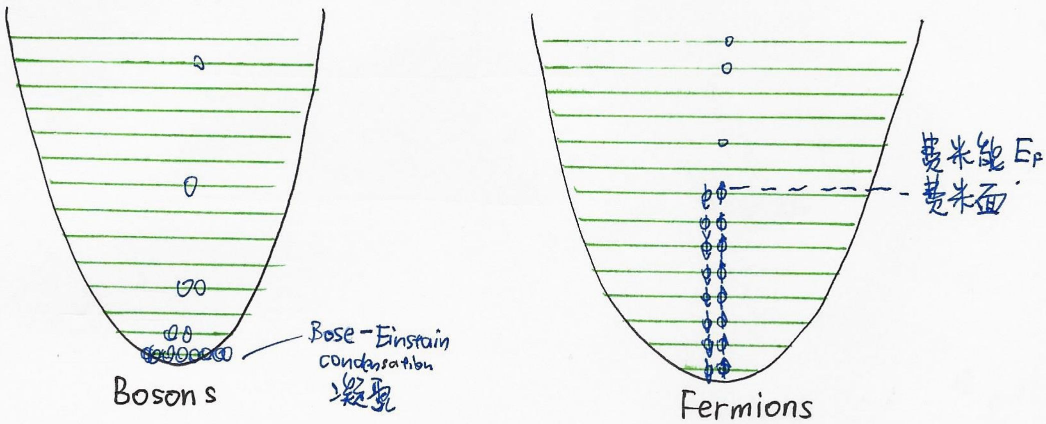
Pauli Exclusion Principle

在一个量子系统中，两个费米子不能拥有完全一致的量子数.

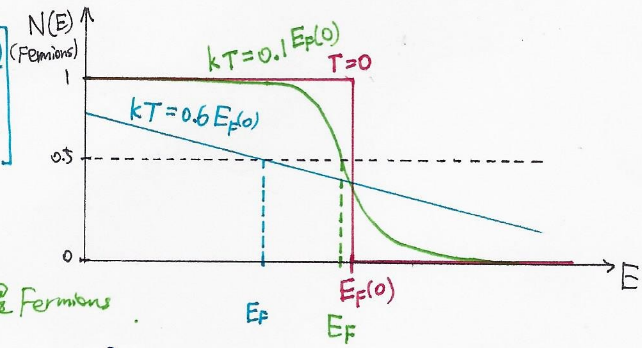
$$\begin{pmatrix}\text{粒子的}\\ \text{统计性}\end{pmatrix}\text{概率}\ N(E)=\begin{cases}\dfrac{1}{Be^{\frac{E}{kT}}} & \text{Boltzmann distribution (经典，粒子可分辨)}\\[2ex] \dfrac{1}{B\left(e^{\frac{E}{kT}}-1\right)} & \text{Bose-Einstein distribution (bosons) int-spin}\\[2ex] \dfrac{1}{B\left(e^{\frac{E}{kT}}+1\right)} & \text{Fermi-Dirac distribution (fermions) half-int-spin}\end{cases}$$







$$\left[\text{量子 or 经典}\right]\ \text{取决于}\left\{\begin{matrix} T \lesssim \dfrac{E_F(0)}{k_B} \\ \text{or} \\ T \gg \dfrac{E_F(0)}{k_B}\end{matrix}\right]$$



注：所有具有静止质量的基本粒子都是 Fermions.

相空间密度 $\rho = n\cdot\lambda_D^3 = n\left(\dfrac{2\pi\hbar}{M k_B T}\right)^{\frac{3}{2}}$ 是无量纲数

$\rho \gg 1$，量子气体　　$\rho \sim 2.6$ Bose gases 开始出现 BEC

$\rho \sim 1$，量子液体　　$\rho \sim 1$ Fermi gases 开始填满低能级.

$\rho < 1$，超流体

Quantum Computing. N qubits $\sim 2^N$ bits.

3 qubits $\sim$ 8 bits

$$\Psi = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + a_3|011\rangle$$
$$a_4|100\rangle + a_5|101\rangle + a_6|110\rangle + a_7|111\rangle.$$

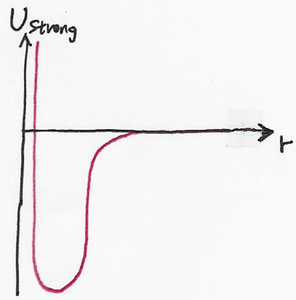
Isotopes 同位素

估算核子大小：卢瑟福散射公式与实验对比.

可测出核的密度几乎为常数. 核物理 —— 流体力学

Force	Relative Strength	Range
Strong (residual) 剩余	1	~ 1 fm.
EM	$\sim 10^{-2}$	∞.
Weak	$\sim 10^{-6}$	$\sim 10^{-3}$ fm.
G	$\sim 10^{-39}$	∞

注：real Strong force 是 quarks & gluons 之间 (核内)
residual 是 proton 与 proton 之间的核力 (不同的核接近)
$p\sim n$, $n\sim n$, …

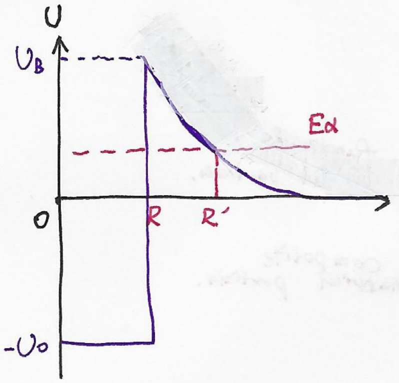


核物理

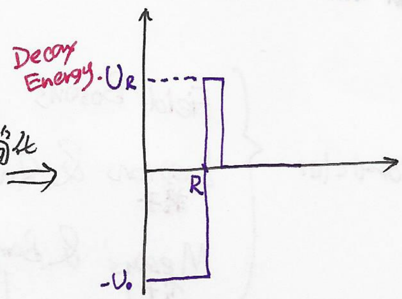
Conservation Laws of
- Energy
- Linear Momentum
- Angular Momentum
- Electric charge.
- Nuclear Number (#p + #n) 高能物理可不遵守

核物理又称"中能物理" 相对

Quantum Theory of α Decay.

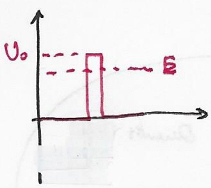


简化 $\Longrightarrow$



half-life 对 Decay Energy 非常敏感 (指数)

结论:



的 Transmission Amplitude 为: $\frac{C}{A} = \frac{-4ik_1k_2e^{-ik_1a}}{(k_1+ik_2)^2e^{k_2a} - (k_1-ik_2)^2e^{-k_2a}}$

$k_1^2 = \frac{2mE}{\hbar^2}$, $k_2^2 = \frac{2m(V_0-E)}{\hbar^2}$

对 V_0, E 非常敏感.

β decay $\quad n \rightarrow p + e^- + \bar{\nu}_e$ anti neutrino

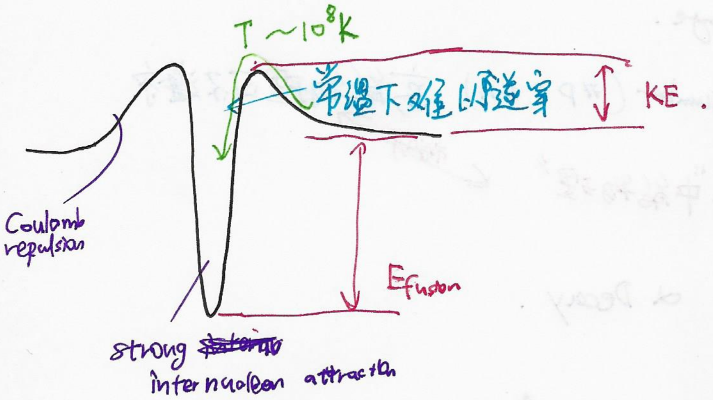
实验上尚未区分 (Majorana fermion?)

anti β-decay $\quad p \rightarrow n + e^+ + \nu_e$ neutrino

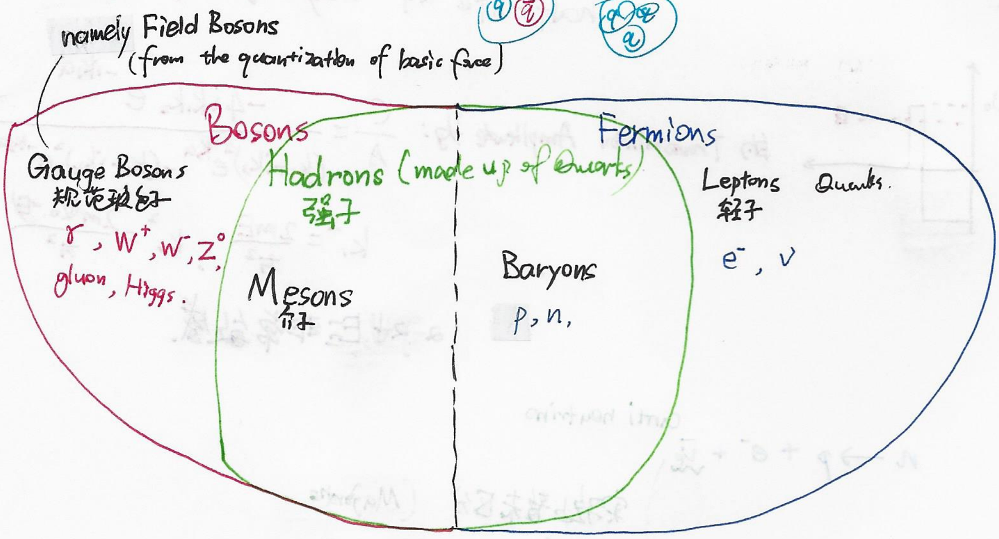
Neutrino
- Electron ν
- Muon ν
- Tau ν

注: 是否衰变可判定粒子有无质量.

Fusion 聚变.



Elementary Particles
- Field Bosons
- Leptons 轻子 & Quarks : fundamental material particles.
- Mesons 介子 & Baryons : composite material particles.



Standard Model

Quarks			Gauge Bosons	
U Up	c charm	t top	g gluon	H Higgs boson
d down	s strange	b bottom	γ photon	G graviton

Leptons			
e electron	μ muon	τ tau	Z Zboson
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson

generation	1st	2nd	3rd.
	Ordinary matter	Cosmic rays	Accelerations.

$\longrightarrow$ rest energy

range of force & mass of mediator.

由 $\Delta E \Delta t \geq \frac{\hbar}{2}$, $\Rightarrow$ range $\sim \Delta x \simeq \frac{\hbar}{c} \cdot \frac{1}{m}$!!!

zero mass ~ inf. range

finite mass ~ finite range